A complete parameter study approach to designing differential bevel gears

Subtitle: Performance prediction for forged bevel gears geometries used in differentials

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Abstract

Modern forged bevel gear geometries widely used in automotive differentials differ strongly from classical machined designs, which limit the accuracy of performance prediction using standard ISO calculation routines. This is mainly related to variable root radius designs, forging related tip geometries and webbing designs with varying tooth height factors at toe and heel.

Through extensive testing and correlation work a simplified calculation could be obtained in the past, however leading to very different designs across car makers for the same vehicle class and general road usage. Although standard ISO tools provide some basic sizing information, they are only used to a limited extent trying to obtain “clean sheet” optimized designs with potentially higher power densities.

State of the art FEA on the other hand allows better analyzing of stress distribution and correlating test results for any existing design. But due to calculation times and the necessity of exact models, this process is not feasible for a wider range parametric analysis.

As part of its strategic product planning process, GKN has challenged this situation and built a project team with company KISSsoft to develop a calculation method combining the best of both worlds – fast multi-parametric variants calculation and a more accurate stress analysis for forged geometries.

Following this method the macro geometry is varied by many parameters such as pressure angle, numbers of teeth, tooth heights, root and face cone angles, profile shift coefficients, tip and root radii, etc. Specific and tailored boundary conditions such as limiting contact pressure or geometric boundaries are used to reduce the huge amount of solutions to a realistic number.

The strength rating itself is based on a modified ISO procedure, whilst the contact analysis is enhanced to reflect the gear shape with webbings and tip alterations and to account for the

specific geometric properties influencing tooth stiffness. Micro geometry modifications with standard values are considered to determine load distribution and hence tooth bending, which results in a most realistic transmission error calculation.

GKN’s ultimate goal is to find a robust optimum in bevel gear macro and micro geometry with minimized packaging for GKN AWD- and eDrive product stream applications (considering new product requirements such as special NVH performance characteristics required by AWD Booster™ disconnect drivetrains or changed durability requirements for eDrive drivetrains) to meet both performance and manufacturability constraints. Being at the heart of our components, differential sizing strongly influences system packaging from inside-out. Any benefits gained here often allow a complete downsizing of surrounding components.

Introduction

GKN’s ultimate goal is to find a robust optimum in bevel gear macro and micro geometry with minimized packaging for GKN’s AWD and eDrive product range whilst meeting both performance and manufacturability constraints. Minimizing packaging of differential bevel gears strongly influences system packaging from inside-out and any benefits gained here often allow a complete downsizing of surrounding components. Additional challenges are given by AWD Booster™ disconnect drivetrains, which require special NVH performance characteristics of their differential bevel gears due to their special running conditions, when disconnected or transitioning between both states connected and disconnected.

Classical vs. modern gear design and gear design process

Modern forged bevel gear geometries (see Fig. 1), which are widely used in automotive differentials, differ strongly from classical machined designs. Forging processes offer advantages like generation of

- variable root radii designs, allowing to optimize tooth root strength and stress distribution over face width
- free form tip geometries, allowing to optimize tooth mesh and contact ratio
- webbing designs accompanying varying tooth height factors at toe and heel, allowing to achieve larger face width and to strengthen toe and heel against stress

But the fact, that modern forged bevel gear geometries differ strongly from classical machined designs, limits the accuracy of performance prediction with standard ISO 10300 [1, 2, 3] calculation routines. The reason for this is that ISO 10300 considers only the virtual cylindrical gear of a bevel gear at middle of face width as reference and doesn’t consider variable root radius or webbing influences on stresses. With these specific geometric features the actual face width changes significantly over profile height, while ISO 10300 only assumes a constant face width. Within ISO 10300 calculations webbings therefore have to be handled by worst, average or best case scenarios, or in other words by assuming a face width that might be averaged or vary from minimum to maximum common face width. Depending on accuracy requirements this can lead to a number of additional calculations, for example by using a small face width to evaluate surface contact stresses at the tip but a larger face width to analyze root bending stresses.

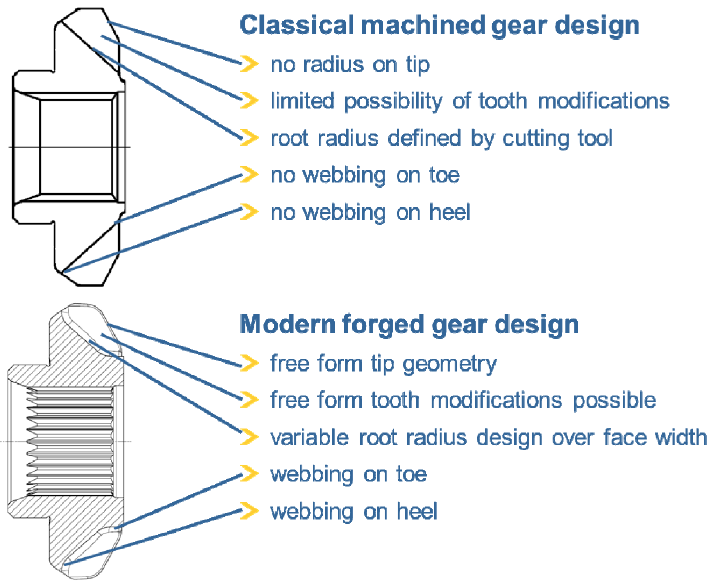

Fig. 1: Comparison of modern forged and classical machined bevel gear geometries

Additionally, webbings cause a change in stiffness at the tooth ends because they connect tooth ends to gear body and stiffen thereby the tooth ends on toe and heel - resulting in changed tooth deformations and pressure distribution under load, which cannot be considered by ISO 10300.

Against this background today's FEA/CAE tools (e.g. product Marc of MSC Software Corporation or Creo® Simulate of PTC®) are state of the art software for prediction and analysis of stress distribution on tooth flank and in tooth root of forged bevel gears, because these tools allow consideration of the exact gear design respectively tooth flank, tooth root, webbings and other gear body geometry parameters due to CAD model interface. In contrast to ISO 10300 calculations CAE analyses don't provide any safety factors. Thus interpretation of CAE results requires correlation with bench or vehicle test results for a multitude of designs in order to generate permissible stress level values for sizing.

As a rough estimation, 1 gear design CAE calculation run lasts, depending on the required number of tooth mesh positions, from 1 to 3 days. Further the investigation of design variants

requires a manual generation of new CAD models, which lasts from some minutes for a minor geometry change (e.g. modified tip radius) up to 1 day for a major design change (e.g. changed numbers of teeth or macro geometry). As a result CAE calculations are not applicable for a wide range parametric analysis to define an optimized gear design.
Because of this today design engineers often define gear designs based on heuristics, thumb rules or internal empirically derived guidelines. Typically the final gear design is found by an iterative procedure (see Fig. 2). Starting point is the investigation of the conditions given by an existing gear design designed for similar load conditions. In several further steps the design engineer tries to optimize the stress conditions on tooth flanks and in tooth roots by stepwise variation of single gear geometry parameters. Today this process is normally supported by analytical or FEA-based software tools, which allow calculating the influence of these gear design modifications on running behavior and loading of the gears. Being very time consuming, this process often takes days or even weeks, and multiple loops, while the quality of the tooth design still strongly depends on the experience and also on mental/physical state of the design engineer. The results are seldom objective in nature and 100% repeatable.

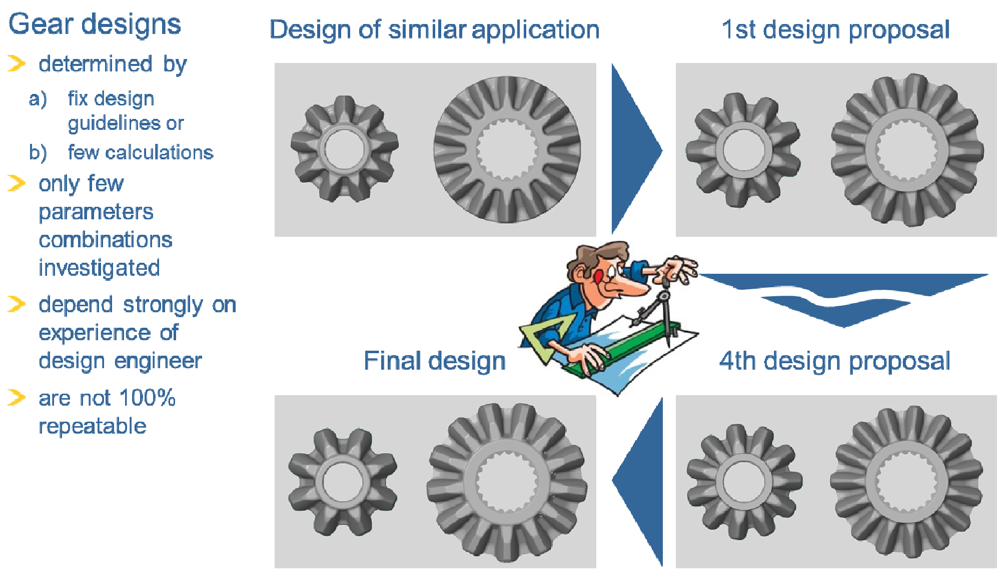

Fig. 2: Determination of gear design – Classical design process

In terms of quality it has to be mentioned that the quality of the generated gear design cannot be rated properly. It is only possible to prove that the best of all investigated gear designs was chosen, but due to the wide parameter range it is not possible to determine whether there is a global optimum better than the found local optimum.

In order to ensure that the gear design chosen for an application is the best or at least close to the best possible gear design GKN has decided to setup and implement a robust strategy by using a full parametric design process (see Fig. 3) to enable the following:

1) Due to clear knowledge about the effect of gear geometry parameter variation it might be possible to allow smaller gear sizes, what leads to reduced differential size, weight and production costs - not only on differential, but on system level.
2) Existing gear sizes might be kept but higher loads could be applied to meet the ever increasing demand in torque density.
3) If the parametric design study considers not only main geometrical parameters like module, face width and numbers of teeth, but also production process related parameters like allowable materials and minimum required tip radii etc., a robust gear design can be found, that can be manufactured by various production processes such as cold and warm forging and uses materials, which are available worldwide. This supports a global availability and standardization strategy with full design ownership, not having to rely on gear forger’s off the shelf designs.

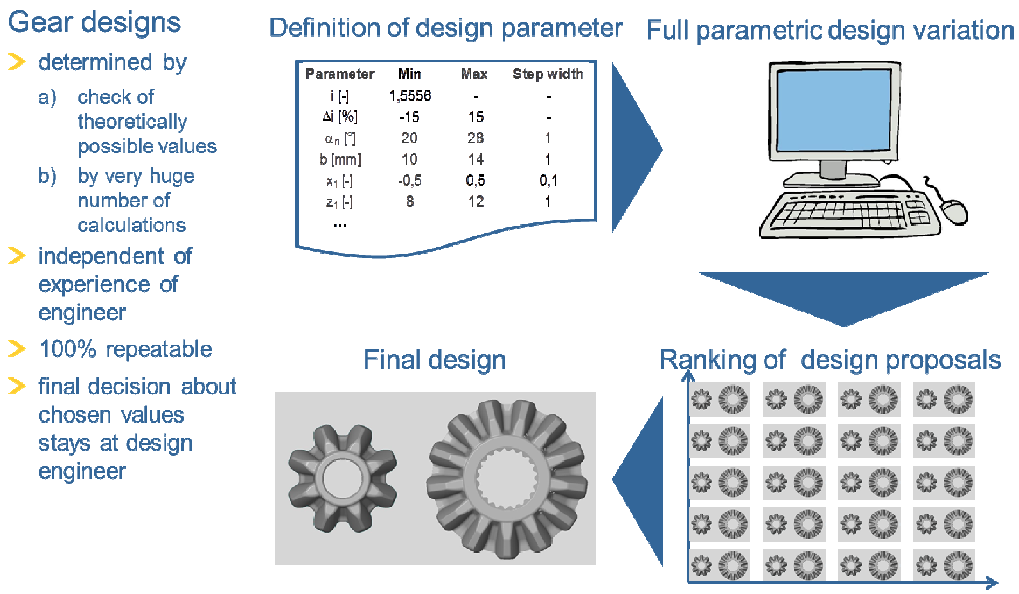

Fig. 3: Determination of gear design – Modern design process implemented by GKN

By following this new process GKN gear designs are determined on basis of a full parametric check of all theoretically possible value ranges of design parameters, while considering certain boundary conditions given by production, material or design space. As described later this causes on the one hand a very huge number of calculations, but is on the other hand independent of the design engineer’s experience and thus repeatable.

This computer aided procedure helps to train inexperienced design engineers and to ensure acceptance criteria optimized designs by guiding the design engineer in an adequate way through the design process of gears, while considering all constraints related to load carrying capacity, noise behavior and production needs.

The final decision about which parameters are to be varied, their ranges, as well as the decision/selection of final gear design stays with the design engineer. This allows rerunning the optimization procedure whenever new sets of input parameters appear on the horizon.

The following describes how this was realized by GKN in cooperation with company KISSsoft.

Realized full parametric gear design process

Starting point of the full parametric gear design process is the definition of load data and geometrical restrictions in KISSsoft software [4]. Initially the user gives input similar to input for standard gear design calculation acc. to ISO 10300 [1, 2, 3] or DIN 3991 [5, 6, 7] (such as load data, material and lubrication conditions as well as additional information about required gear quality and backlash). Additionally he has the possibility to define an existing design for investigated application, which can later be used as reference for new found gear designs (see Fig. 4).

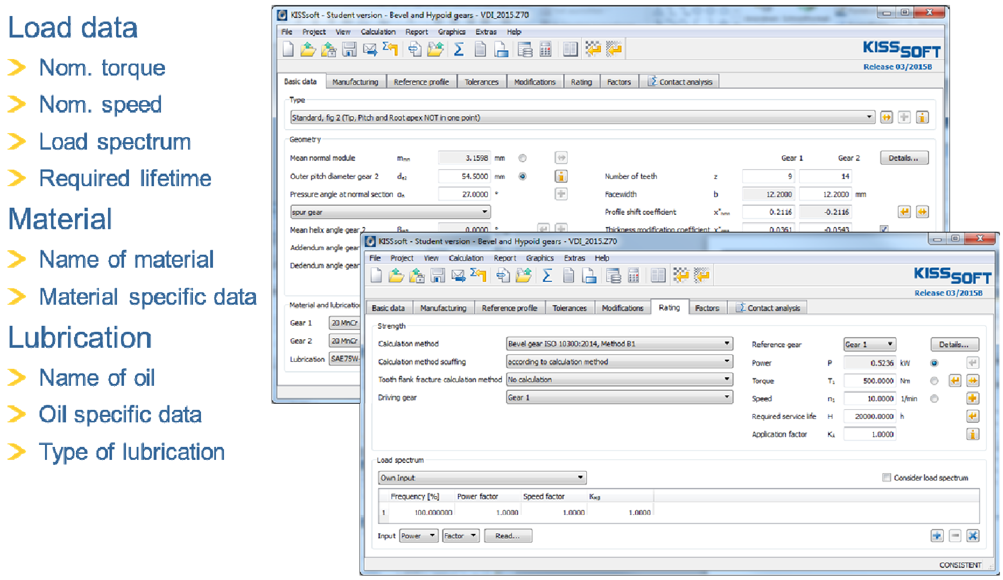

Fig. 4: Input of load data, material and lubrication

After this input is given in 2 further menus for various gear geometry parameters allowable ranges and their step widths can be specified (see Fig. 5). Finally, various geometrical restrictions are specified, which have to be considered in the full parametric gear design process. The range of potential geometrical restrictions was extended acc. to needs of forged bevel gears in differentials (see Fig. 6).

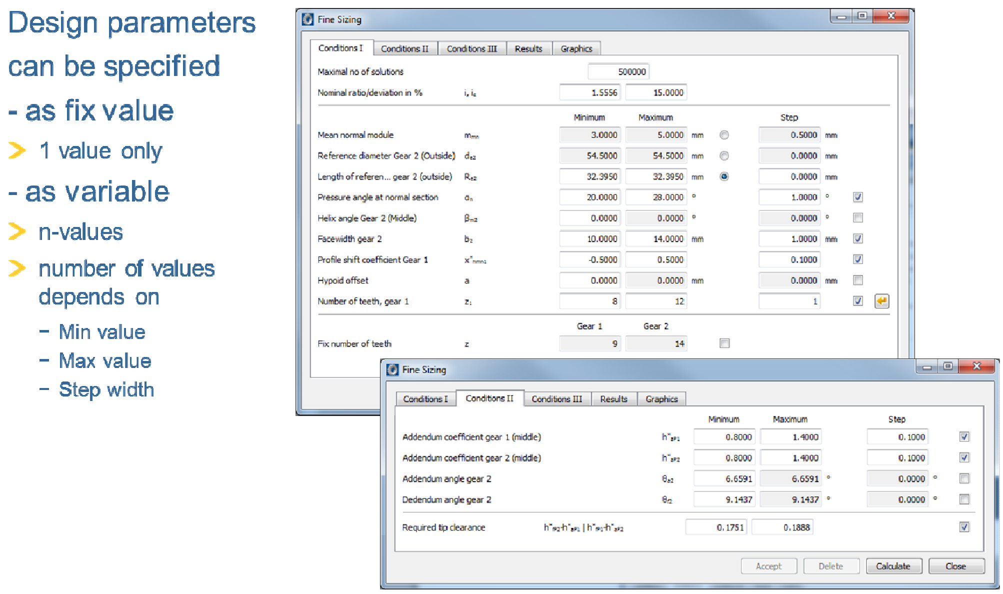

Fig. 5: Geometrical parameter used for gear design definition in full parametric gear design process

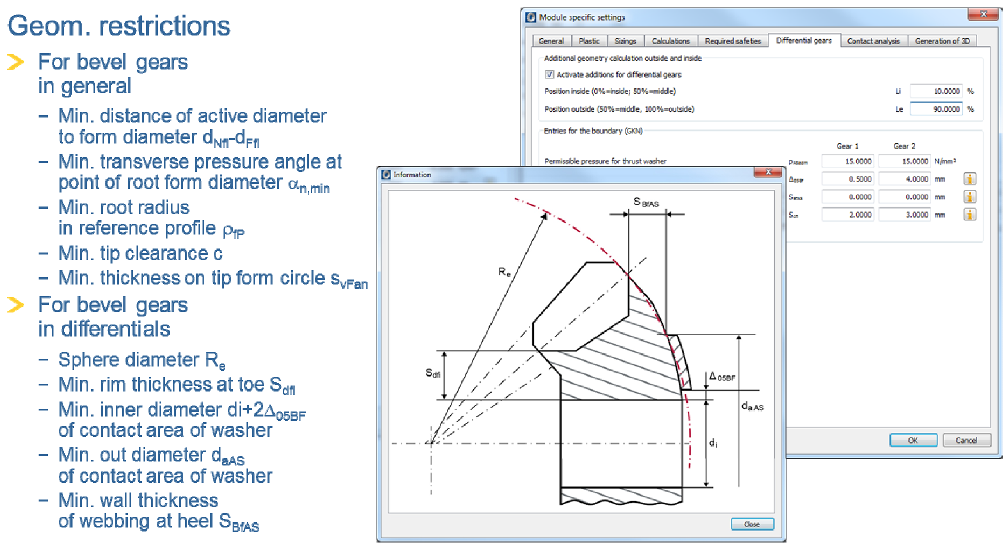

Fig. 6: Input of geometrical restrictions

Based on this user input, KISSsoft generates all possible combinations of given gear geometry parameters and checks automatically whether these values are applicable or whether they have to be reduced. As before e.g. tooth height gets reduced automatically if at actually investigated tooth height the minimum required tip radius can't be realized. These checks are preformed not only in middle of face width but also in user given positions at inner and outer end of face width.

But also new constraints are considered now. With regards to gear body geometry it gets checked whether a minimum required hoop thickness around bore of gear is given or whether face width has to be reduced to realize required hoop thickness. If so, also the mating gear gets automatically adjusted accordingly in order to prevent gears from jamming or interference. The same is done on tooth root if KISSsoft detects that tooth root has to be adjusted in order to realize sufficient thickness of gear body between tooth root and back face of gear body.

In this context it has to be mentioned that KISSsoft checks automatically for each parameter variation, whether actually combined parameters define an applicable gear design. If given geometrical constraints (see Fig. 4, above) are in conflict with an individual design this design gets rejected automatically. This check means high comfort for design engineers, because often the consideration of geometrical constrains affects quite heavily a promising, not yet geometrically checked classical gear design that it has to be rejected, e.g. because of too low strength. In practice this means that sometimes only a few hundred gear designs can be found, even if several ten thousands were investigated.

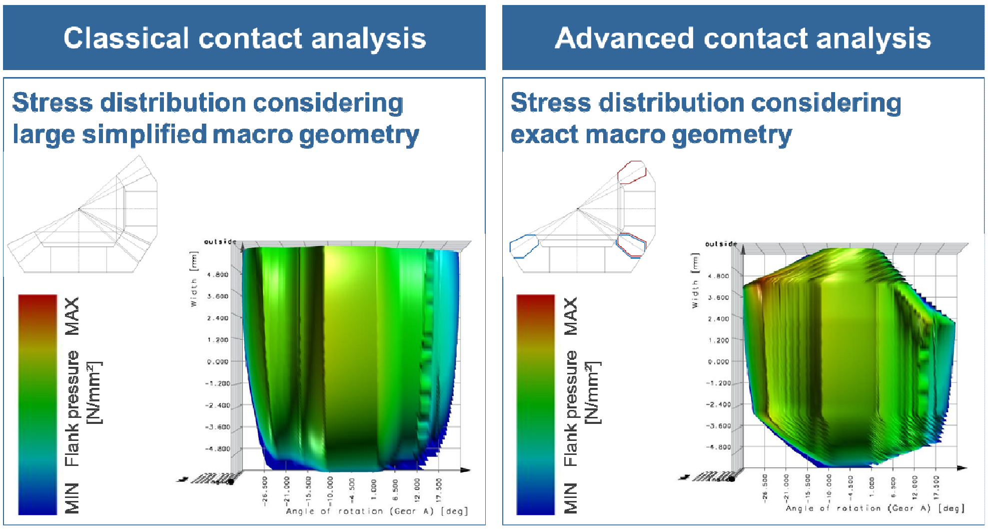

Fig. 7: Contact analysis, simplified vs. exact gear design

For each of the so found geometrical solutions, standard state of the art calculations acc. to DIN 3991/ISO 10300 are performed automatically, whereat only a simplified gear design based on the tooth form in the middle of face width is taken into consideration. In addition, KISSsoft Release 2015 offers now automatic, detailed contact analyses (see Fig. 7) on user demand for all found design solutions, which consider exact gear design inclusive all webbings and cut off areas of tooth flank providing the flank pressure and root stresses considering the real tooth shape. This allows for the first time to perform a full parametric design process and to rate strength of found forging specific gear designs rapidly.

Number of variants vs. runtime

On the one hand engineers in general intend to investigate technical issues with highest possible resolution. On the other hand the number of calculations to be performed defines the runtime of the software.

Considering the number of parameters and the number of variations per parameter the parameter matrix has to be set up carefully. The size of the parameter matrix is determined by the product of number of parameters and number of variations per parameter. Due to this multiplicative character of parameter matrix size, an increasing number of parameters leads quickly to an extremely high number of necessary calculations and thus to a high runtime of the software.

Parameter	# Variants [-]	Total # Calculations [-]	Total CPU Time [hh:mm:ss]	# Variants [-]	Total # Calculations [-]	Total CPU Time [hh:mm:ss]
α_n	5	5	00:00:00	9	9	00:00:00
b	3	15	00:00:00	5	45	00:00:00
x_1	6	90	00:00:01	11	495	00:00:05
z_1	5	450	00:00:05	5	2 475	00:00:27
z_1 / z_2	4	1 800	00:00:20	4	9 900	00:01:50
h^*_{ap1}	4	7 200	00:01:20	7	69 300	00:12:49
h^*_{ap2}	4	28 800	00:05:20	7	485 100	01:29:46

Table 1: Number of variants vs. runtime, calculation acc. to DIN/ISO only, performed on PC with Intel® Core™ i5 CPU @ 2.60GHz and 8GB RAM

Table 1 compares the total number of calculations, when each parameter is investigated in a value specific “standard” step width and when “standard” step width gets reduced by 50%. The overview shows, that the total runtime needed for the investigation of a parameter matrix with 7 parameters, which vary in standard step width, is about 5:20 minutes, if only calculations acc. to DIN/ISO (and no contact analysis) are performed. This is a very comfortable computing time, because now the automatically performed full parametric investigation of a solution space with 28.800 variants takes nearly the same time as a software user would need for manual input of a single new variant. Even an optional eighth parameter would lead to an acceptable runtime of less than 1 hour.

If the “standard” step width gets reduced by 50% whilst parameter ranges stay the same, the number of variants per parameter nearly doubles. In the given example the total number of variants increases by factor ≈17. This results in a runtime of about 1:30 hours, which is still acceptable but shows clearly that the numbers of geometry parameters and their variations have to be chosen carefully.

With regards to runtime behavior, it has to be taken into account that the use of contact analysis increases runtime significantly by factor 120, compared to runtime if only DIN/ISO calculations are performed. While a calculation acc. to DIN/ISO lasts about 0,011 seconds a calculation incl. contact analysis lasts in average about 1,386 seconds.

Table 2 shows that an investigation of 28.800 design variants performed with “standard” step width and contact analysis would last about 11:10 hours instead of 5:20 minutes, if only DIN/ISO calculations were performed. In order to shorten this response time, all potential solutions get checked in terms of geometrical constraints first. Only such solutions, which fulfil all geometrical constraints, are investigated by contact analysis. The right part of Table 2 shows an example where it was possible to reduce response time from 11 hours to 30 minutes by this means.

Parameter	# Variants [-]	Total # Calculations [-]	Total CPU Time [hh:mm:ss]	# Variants [-]	Total # Calculations [-]	Total CPU Time [hh:mm:ss]
α_n	5	5	00:00:07	all variants fulfilling given geometrical constraints	1 316	00:30:24
b	3	15	00:00:21			
x_1	6	90	00:02:06			
z_1	5	450	00:10:29			
z_1 / z_2	4	1 800	00:41:55			
h^*_{ap1}	4	7 200	02:47:39			
h^*_{ap2}	4	28 800	11:10:37			

Table 2: Number of variants vs. runtime, calculation acc. to DIN/ISO & contact analysis, performed on PC with Intel® Core™ i5 CPU @ 2.60GHz and 8GB RAM

Such opportunities to save CPU time decrease the more realistic solutions are found in investigated solution space. Thus it is strongly recommended to scan potential solution space for areas of parameter combinations, which fulfill design targets in best manner, by a stepwise zooming-in scan procedure.

Scanning and ranking of results

In GKN this scanning process was realized by exporting KISSsoft calculation results into text files with table format. These are imported in table calculation software (e.g. Microsoft Excel), where they are ranked mathematically according to actual GKN engineering philosophy, e.g. in terms of minimized tooth flank pressure, root stress and/or transmission error and others. Ranked solutions are sorted with regard to one or several of the rankings. Solutions that do not fulfill required limits of ranking are sorted out. The upper and lower limits of geometrical parameters, which remain in a revised database, are used for the next step of full parametric investigation.

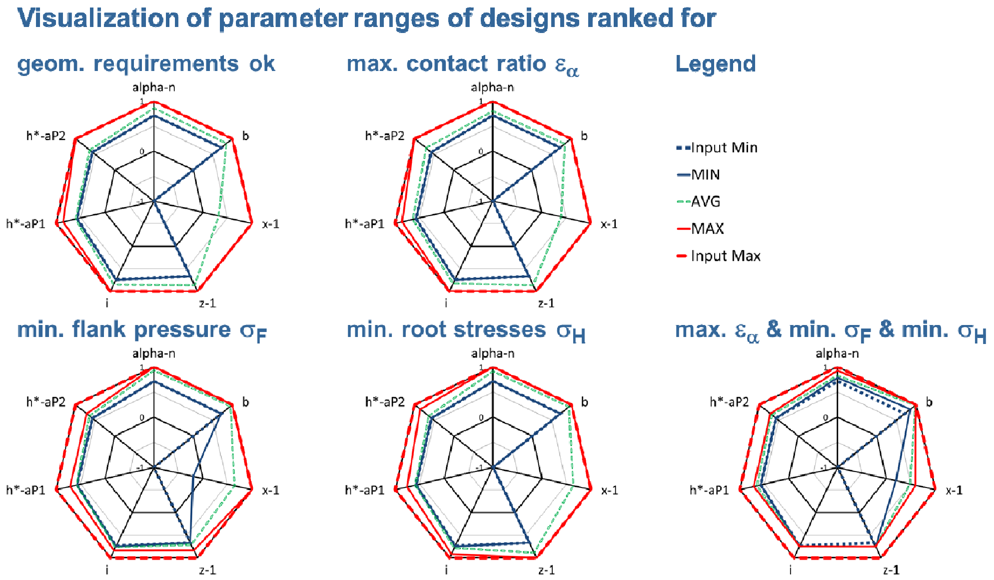

Fig. 8: Scanning and ranking of results

Fig. 8 shows an example for reduction of parameter space by consideration of single and combined rankings. In this case the upper left diagram shows that design parameters of design variants, which fulfill all geometrical requirements, are spread over the entire software user given parameter ranges. Only the range of addendum coefficient h^*_{aP1} could not be used entirely. In the upper middle diagram similar information is given for parameter ranges of designs that fulfill all geometrical requirements and offer additionally maximum values for contact ratio ε_α. Here the graph of average parameter values indicates that these designs differ from the earlier shown. More impact on usable design parameter ranges can be detected for designs that fulfill all geometrical requirements and offer additionally minimum values for root stresses σ_F or flank pressure σ_H. Finally, the diagram for designs, which fulfill all geometrical requirements and offer additionally both maximum values of contact ratio as well as minimum values for root stresses σ_F or flank pressure σ_H, shows very narrow design parameter ranges. These ranges can be used for a next design parameter scan. To perform such scanning and ranking several times in sequence allows finding a design of high robustness related to its design targets and ensures that the finally chosen design is verifiable one of best possible solutions.

Summary and Outlook

GKN has successfully generated a new generic design process that allows developing forging specific bevel gear designs used in differentials according to defined design and performance criteria. This was made possible by KISSsoft AG by introducing customized software modules that allow adjusting gear design acc. to differential bevel gear specific requirements on geometry and by offering both processes, the strength calculation acc. to ISO as well as the fast contact analysis considering the real tooth shape of bevel gears with forging specific tooth design.

The next step is to combine the wide design parameter variation of gear macro geometry with a wide design parameter variation of gear micro geometry. Therefore similar methods are in progress to define optimized micro geometries for given macro geometry. They are expected to be coming soon.

These software tools will help to find a robust optimum in bevel gear macro and micro geometry with minimized packaging for GKN's AWD and eDrive product stream applications - including special NVH performance characteristics required by AWD Booster™ disconnect drivetrains – to meet both performance and manufacturability constraints.

Literature

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