Bevel gear strength and life rating – the appropriate combination of rating standards with tooth contact analysis

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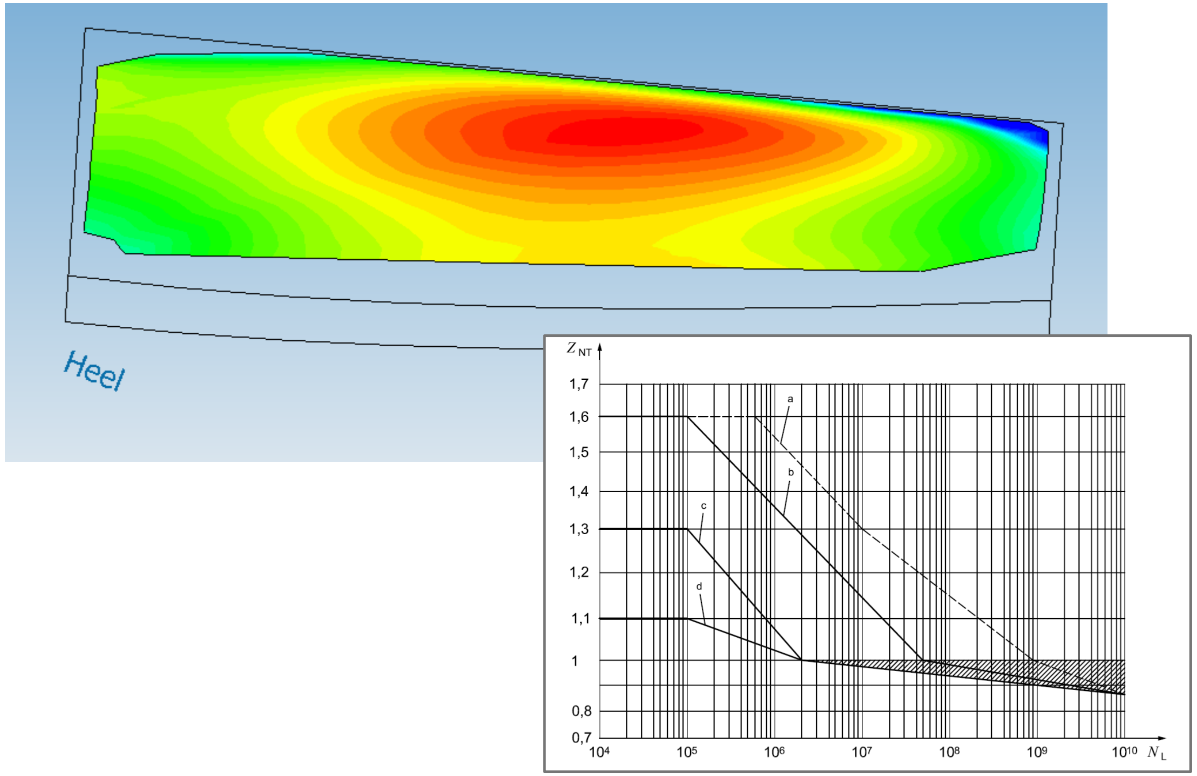

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Overview

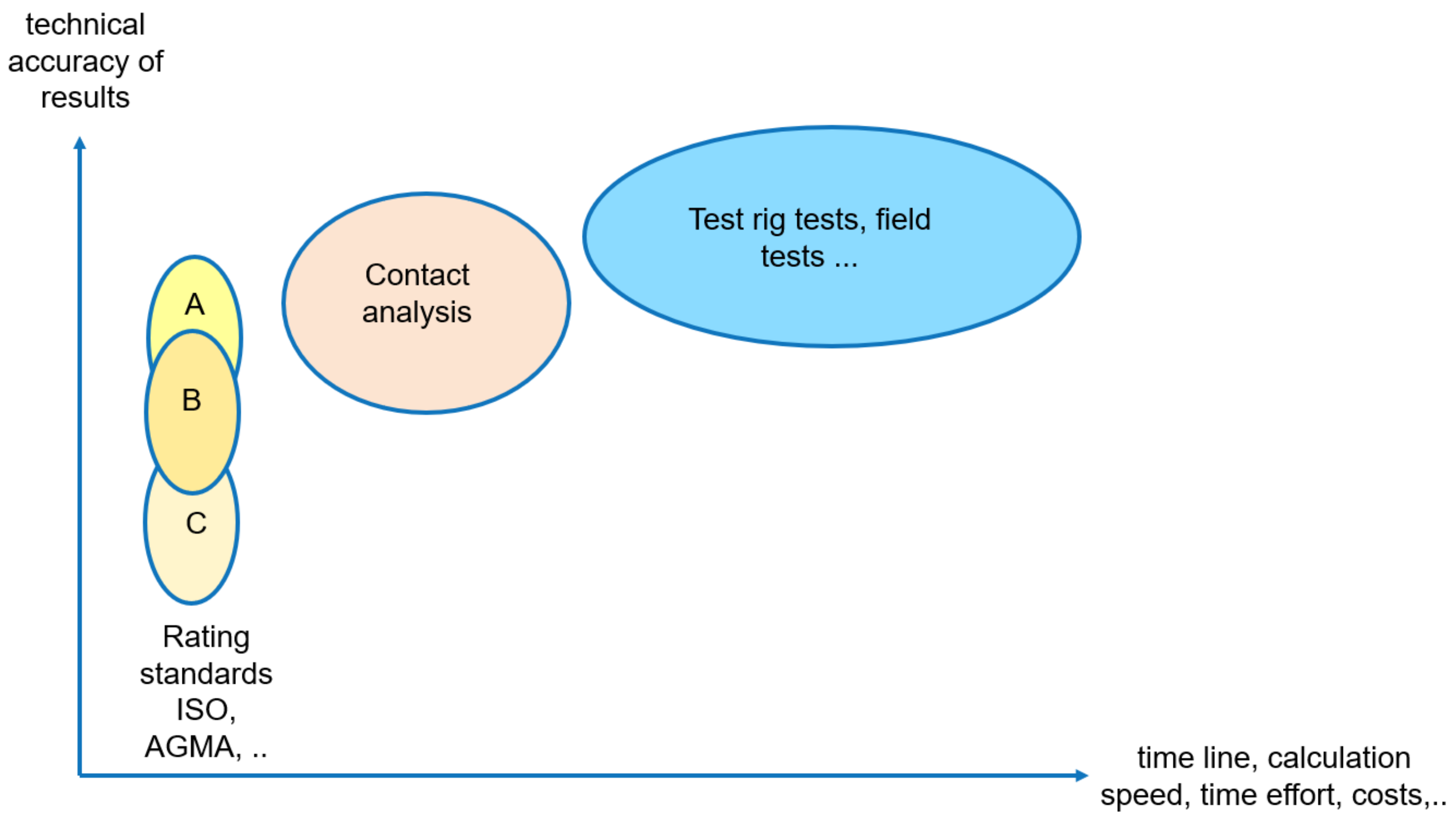
1. Introduction
2. Strength rating methods for bevel gears
3. ISO standard and adaptation possibilities
4. Sample design and calculation of bevel gearset
5. Comparison of stresses
6. Load spectra and damages
7. Conclusion

1. Introduction

For rating of bevel gears, several methods are available, such as rating standards and loaded tooth contact analysis (LTCA).

These differ in available result types, level of detail, accuracy and calculation time.

Typically they are applied in various phases of the design, e.g. variant calculation during dimensioning or development of contact pattern with LTCA.



2. Strength rating methods

The rating standards can provide both, the stress numbers as well as the permissible stress numbers. Herewith, the safety numbers are available.

A disadvantage is the simplified approach, which does not consider micro geometry and assembly misalignments.

The tooth contact analysis (LTCA) takes the exact flank topology into account which is generated during the manufacturing process.

A principal lack of FEM is the missing calculation of permissible stress, and rating of safety and lifetime is not available. GEMS® uses an advanced approach combining FEA and evaluation of lifetime rating.

	Rating standard	Contact analysis	Test rig tests, field tests
Source	ISO, AGMA, ...	FEM, ..	Customer site
Life time information, safety value	✓	✗ (stresses, stress distribution)	✓
Macro geometry	✓ (varied, optimization)	✓ (defined, 1 gear pair)	✓ (defined)
Micro geometry (incl. tolerances & misalignments)	✗ (method A)	✓ (varied, optimization)	✓ (defined)

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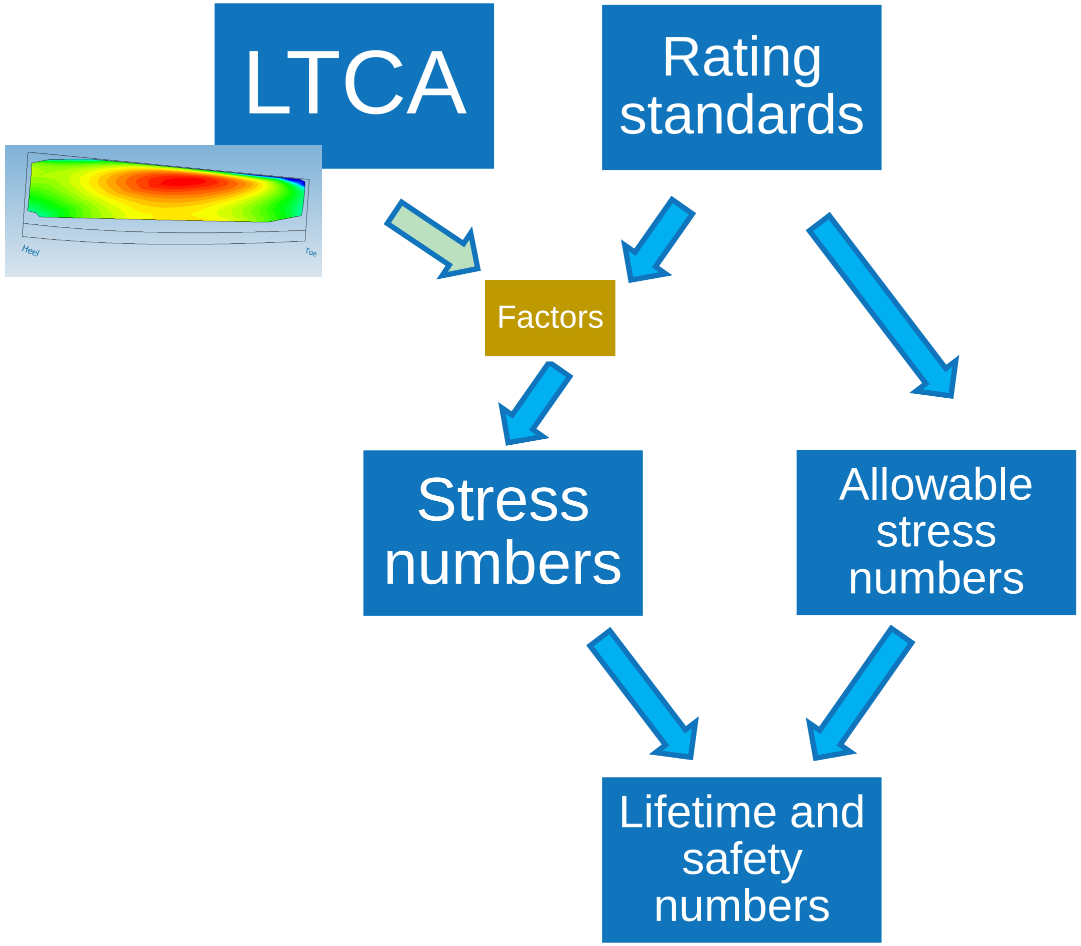
2. Strength rating methods – Combined approach

To have the most precise rating of bevel gearset, the benefit of both calculation approaches should be combined.

The stress numbers of the rating standard should be modified by the results of the LTCA.

Together with the allowable stresses from the rating standard, the resulting lifetimes and safety factors are calculated most precisely.

The rating standards provide some adaptation possibilities to tune the major factors.



3. ISO standard and adaptation possibilities

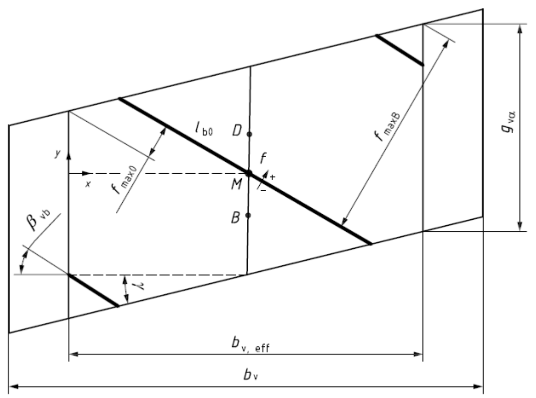
Virtual cylindrical gear and adaptations

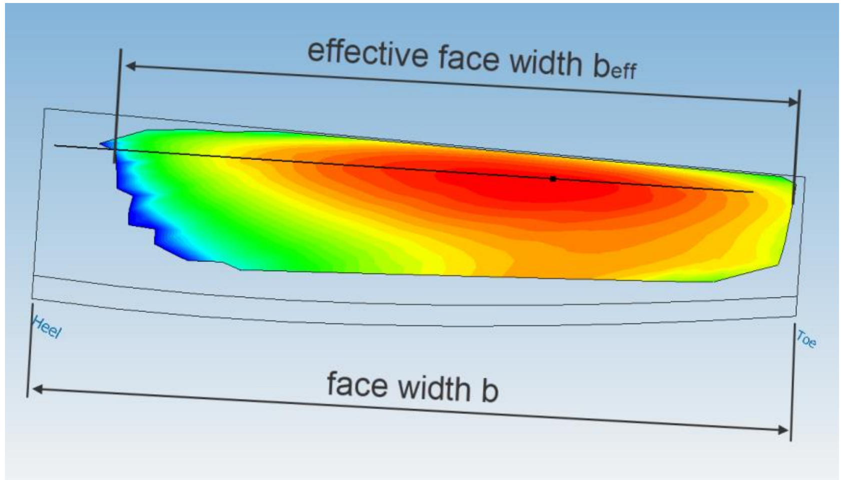
Target is to calculate the contact pattern including the contact lines, simulating the real contact of the bevel gear set.

The shortened contact pattern length is considered with the parameter ‘effective face width’ b_{eff}.

The ISO standard says:

It should be derived from measurements or TCA, at the preliminary design stage $b_{2,\text{eff}} = 0{,}85\ b_2$ is a reasonable estimate.





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3. ISO standard and adaptation possibilities

Surface durability (edition 2014)

The bevel gear factor Z_K is an empirical factor and stress adjustment constant to consider the differences between bevel and cylindrical gears.

$$\sigma_{\mathrm{H0\text{-}B1}} = \sqrt{\frac{F_{\mathrm{n}}}{l_{\mathrm{bm}}\rho_{\mathrm{rel}}}} Z_{\mathrm{M\text{-}B}} Z_{\mathrm{LS}} Z_{\mathrm{E}} Z_{\mathrm{K}}$$

This allows the rating of bevel gears, using the same (resp. slightly modified) rating formulas as for cylindrical gears.

$$\sigma_{\mathrm{H\text{-}B1}} = \sigma_{\mathrm{H0\text{-}B1}} \sqrt{K_{\mathrm{A}} K_{\mathrm{v}} K_{\mathrm{H}\beta} K_{\mathrm{H}\alpha}}$$

The ISO standard (2014) says:

reasonable approximation is given with $Z_K = 0{,}85$

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3. ISO standard and adaptation possibilities

Surface durability (draft for next edition)

The feedback from industry was, that the contact stress from ISO standard is too low.

In the next edition of ISO 10300-2, it is planned to modify the equations.

Calculation of contact stress: without Z_K

$$\sigma_{H0-B1} = \sqrt{\frac{F_n}{l_{bm} \cdot \rho_{rel}}} \cdot Z_{M-B} \cdot Z_{LS} \cdot Z_E$$

To maintain the safety number, the permissible contact stress is increased as well.

$$\sigma_{HP-B1} = \sigma_{H,lim} \cdot Z_{NT} \cdot Z_X \cdot Z_L \cdot Z_v \cdot Z_R \cdot Z_W \cdot Z_{KP} \cdot Z_{Hyp}$$

Calculation of permissible contact: Z_{KP}=1,2

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3. ISO standard and adaptation possibilities

Application factor K_A

The application factor K_A is defined as the ratio between the cyclic peak torque and the nominal rated torque. Nowadays, the factor is often replaced by a load spectrum (duty cycle).

Working characteristics of the driving machine	Working characteristics of the driven machine			
	Uniform	Light shocks	Medium shocks	Heavy shocks
Uniform	1,00	1,25	1,50	1,75 or higher
Light shocks	1,10	1,35	1,60	1,85 or higher
Medium shocks	1,25	1,50	1,75	2,00 or higher
Heavy shocks	1,50	1,75	2,00	2,25 or higher

[a] This table is for speed-decreasing drives only.

For speed-increasing drives, a value of $0{,}01 \cdot u^2$ should be added to K_A, where $u = z_2/z_1$ = gear ratio.

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3. ISO standard and adaptation possibilities

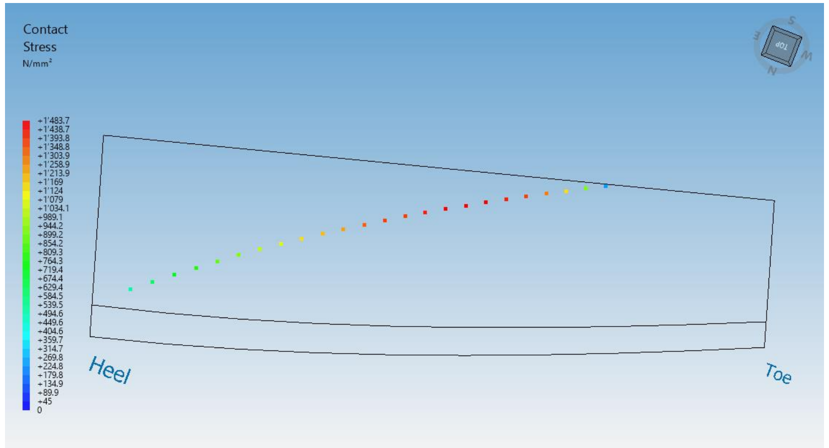
Face load factor $K_{H\beta}$

The face load factor $K_{H\beta}$ reflects the non-uniform distribution of the load along the face width. It is based on the mounting factor $K_{H\beta\text{-be}}$, multiplied by 1.5.

Verification of contact pattern	Mounting conditions of pinion and wheel		
Contact pattern is checked:	Neither member cantilever mounted	One member cantilever mounted	Both members cantilever mounted
for each gear set in its housing under full load	1,00	1,00	1,00
for each gear set under light test load	1,05	1,10	1,25
for a sample gear set and estimated for full load	1,20	1,32	1,50
NOTE Based on optimum tooth contact as evidenced by results of a contact pattern test on the gears in their mountings.			

$$K_{H\beta-C} = 1{,}5\ K_{H\beta-be}$$

By using an LTCA, the load distribution along the contact line can be determined precisely. However, it is not in exact face width direction.



Contact line calculated by LTCA

By evaluating the LTCA results, the load distribution is typically in a range of 1.5 to 1.7.

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3. ISO standard and adaptation possibilities

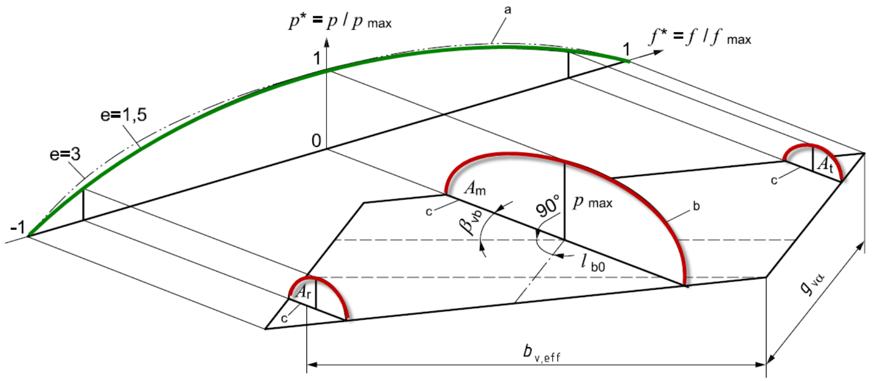
Load sharing factor Z_{LS}

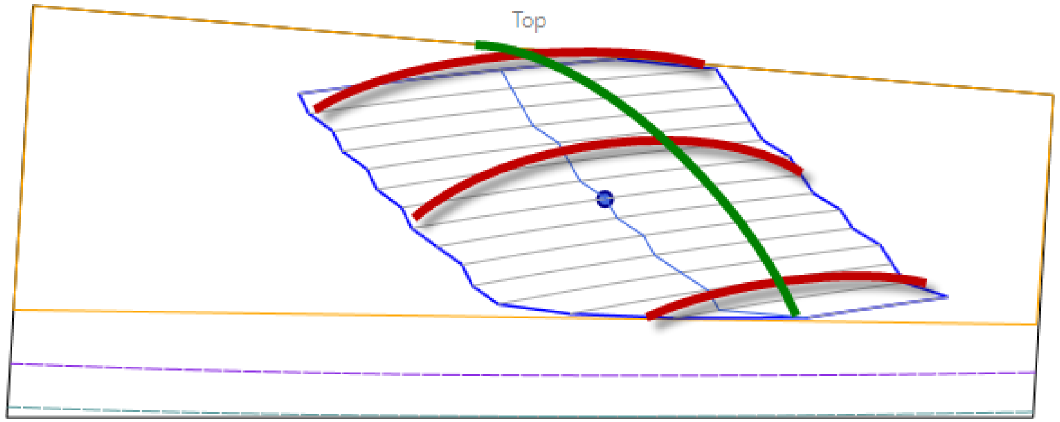
The load sharing factor Z_{LS} considers the load distribution along the path of contact.

As the path of contact goes mainly in profile direction, the factor Z_{LS} represents the amount of profile crowning.

The distribution of the peak loads per each contact line (red lines) along the path of contact (green line) is assumed to follow a parabola.

This parabola can have two distributions, ‘low (e.g. automotive gears)’ and ‘high (e.g. industrial gears)’.



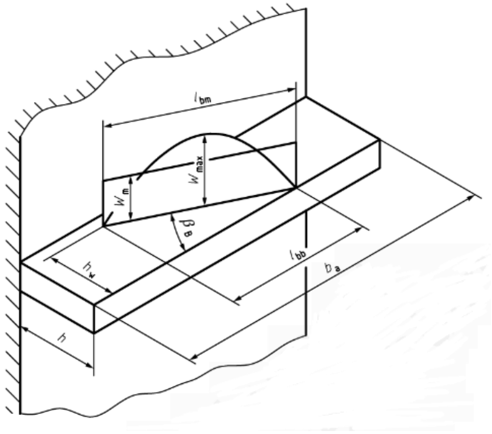


3. ISO standard and adaptation possibilities

Root bending stress

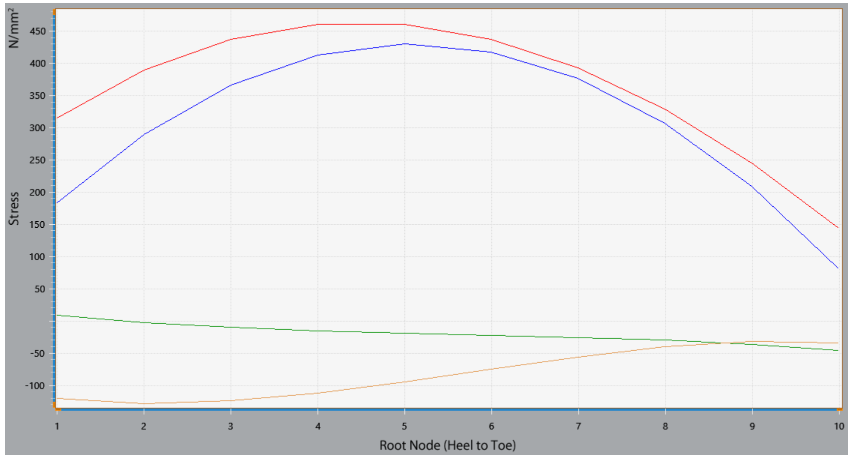
$$\sigma_{F0\text{-}B1} = \frac{F_{vmt}}{b_v\, m_{mn}} Y_{Fa} Y_{Sa} Y_{\varepsilon} Y_{BS}\, Y_{LS}$$

$$\sigma_{F\text{-}B1} = \sigma_{F0\text{-}B1} K_A\, K_v\, K_{F\beta}\, K_{F\alpha} < \sigma_{FP\text{-}B1}$$



The bevel spiral angle factor Y_{BS} accounts for smaller values for contact lines l_{bm} compared to the total face width and the inclined lines of contact.

The bevel spiral angle factor replaces the bevel gear factor Y_K from edition 2001.



With LTCA, the stress distribution is calculated based on the inclination of the contact line and the exact line load along the contact line.

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3. ISO standard and adaptation possibilities

Allowable stresses

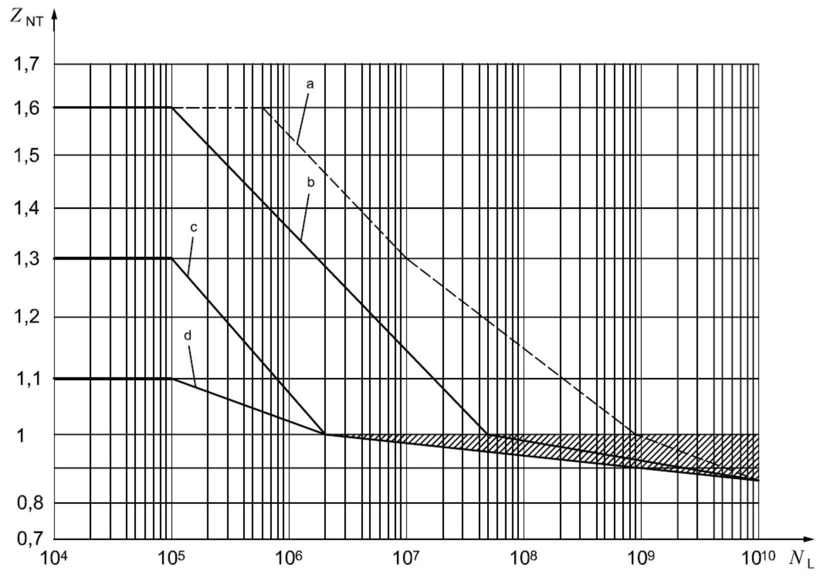
$$\sigma_{HP\text{-}B1} = \sigma_{H,lim}\, Z_{NT} Z_X Z_L Z_v Z_R Z_W Z_{Hyp}$$

$$\sigma_{FP} = \frac{\sigma_{F\,lim} \cdot Y_{ST} \cdot Y_{NT}}{S_{F\,min}} \cdot Y_{\delta\,rel\,T} \cdot Y_{R\,rel\,T} \cdot Y_X$$

For the extended life, the ISO standard provides two options:

A) Z_{NT} resp. Y_{NT} = 1 from the ‘knee’ to 10^{10} load cycles
Applicable for optimum lubrication, material, manufacturing and experience.

B) Z_{NT} resp. Y_{NT} is reduced to 0.85 at 10^{10} load cycles
May be used for critical service, where tooth failures shall be minimal.



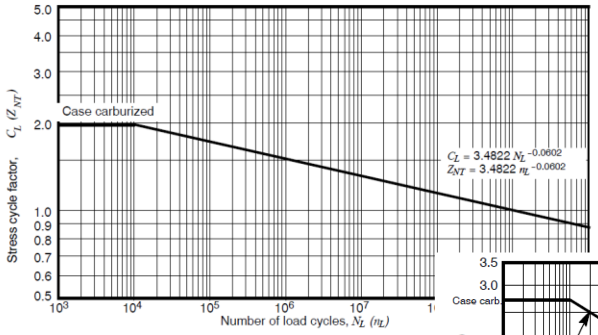
Wöhler lines – which is the right one?

Comparing the Wöhler lines of the typically used standards, the differences become obvious:

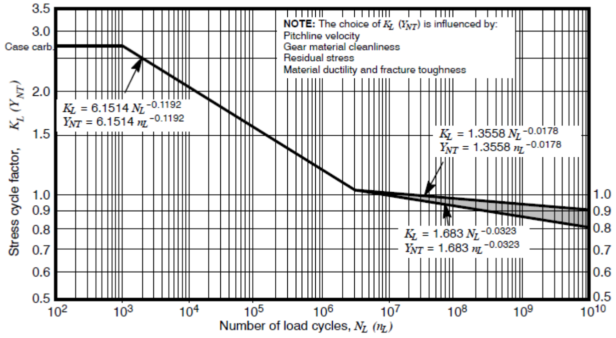
The ISO 10300 has two options. Once for ‘critical service’, which has 0.85 at 10^{10} or ‘horizontal’.

The AGMA 2003 has for root bending stress the knee, having two options (0.8 or 0.9 at 10^{10}) and for contact stress the constant decrease.

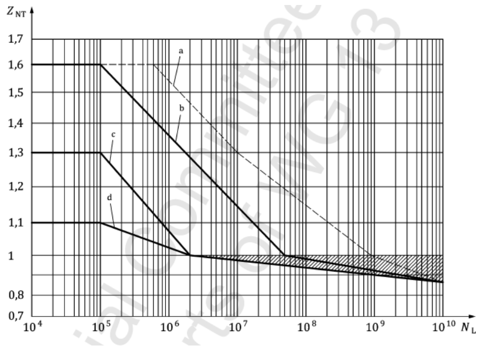
Many designs have load cycles in the area of $10^8 < n_L < 10^{10}$. The Wöhler line has a <u>major impact</u> on the safety number and is therefore to be selected very carefully.



AGMA 2003,
contact stress



AGMA 2003,
root stress



ISO 10300,
contact stress and
root stress

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4. Sample design - Transmission

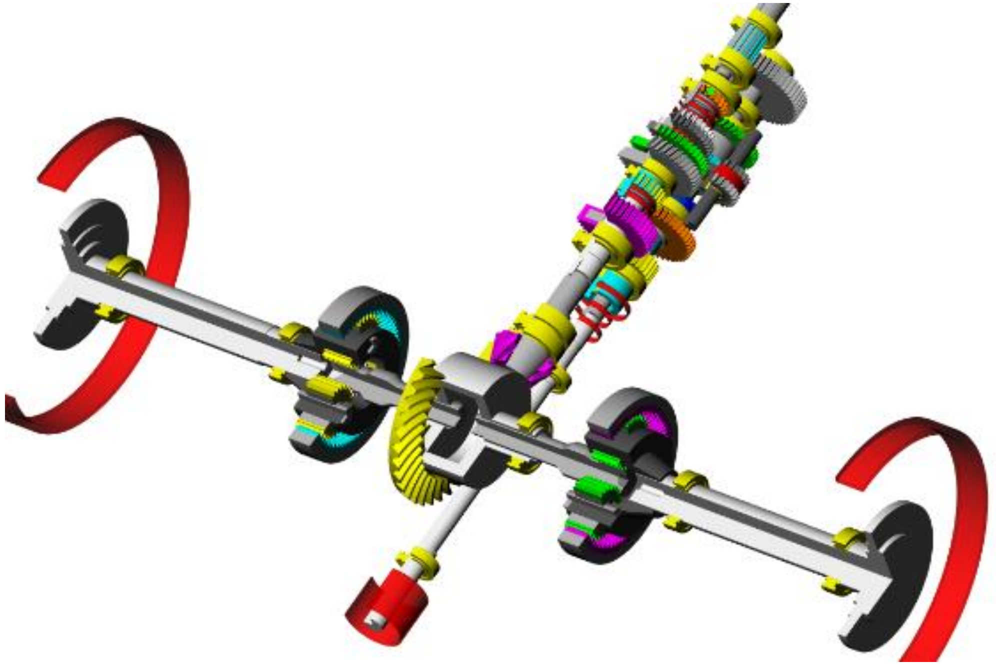
As a sample design, a bevel gearset of a tractor rear axle is used. The drivetrain is modelled in KISSsys and contains the complete drivetrain, including the 8 speeds.

The bevel gearset of the rear axle was designed using an outer ring gear diameter of 310 mm and a ratio of 11:35.

Two load scenarios are calculated:

A) Nominal load with 1500 Nm and 165 rpm at pinion.

B) Load spectrum (duty cycle) with 36 load cases for all 8 speeds, with maximum torque of 1500 Nm at pinion.



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4. Sample design – Misalignment values

Under load, pinion and ring gear are misaligned due to the reaction forces and stiffness of the shafts, bearings and the housing structure.

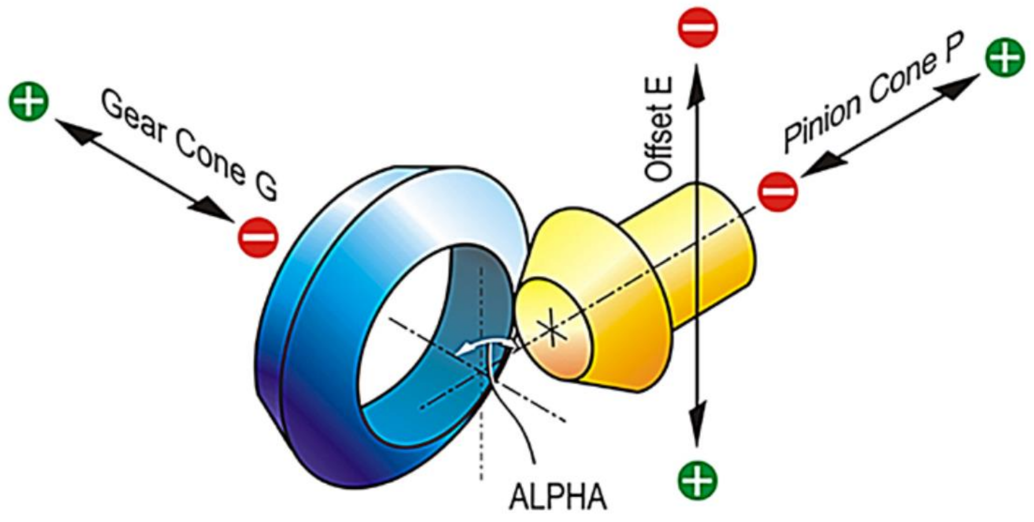
The misalignments are defined by the four parameters E, P, G and Alpha, which are accumulated from pinion and ring gear deflections:

E: misalignment in offset direction

P: misalignment in pinion axis direction

G: misalignment in ring gear axis direction

Alpha (S): angular misalignment in the plane of shaft axis angle



$$\mathbf{S} = a\cos\left(\frac{\bar{v}_1 \bullet \bar{v}_2}{\|\bar{v}_1\| \cdot \|\bar{v}_2\|}\right) \cdot \frac{180}{\pi} - \Sigma \qquad \mathbf{E} = \frac{\left|(\bar{x}_1 - \bar{x}_2) \bullet (\bar{v}_1 \times \bar{v}_2)\right|}{\|\bar{v}_1 \times \bar{v}_2\|} - a$$

$$\frac{\left\|(\bar{x}_{p1} - \bar{x}_2) \times \bar{v}_2\right\|}{\|\bar{v}_2\|} = E + a \quad \text{, where } \bar{x}_{p1} = \bar{x}_1 + (P_{nom} + \mathbf{P}) \cdot \bar{v}_1$$

$$\frac{\left\|(\bar{x}_{p2} - \bar{x}_1) \times \bar{v}_1\right\|}{\|\bar{v}_1\|} = E + a \quad \text{, where } \bar{x}_{p2} = \bar{x}_2 + (G_{nom} + \mathbf{G}) \cdot \bar{v}_2$$

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4. Sample design – Misalignment values

In the drivetrain design software, the misalignment values E, P, G and Alpha are calculated for the torque on pinion T_1 = 1’500 Nm.

Drive side: E = -0.3 mm, P = 0.14 mm, G = -0.23 mm

Coast side: E = 0.41 mm, P = -0.17 mm, G = 0.50 mm

InterfaceGEMS

	Frequency	Torque	Speed	Name	E_mm	P_mm	G_mm	S_deg
1	0.5	1	1	Drive	-0.30038	0.14325	-0.23413	0.086954
2	0.5	-1	-1	Coast	0.41188	-0.17392	0.50156	-0.087613

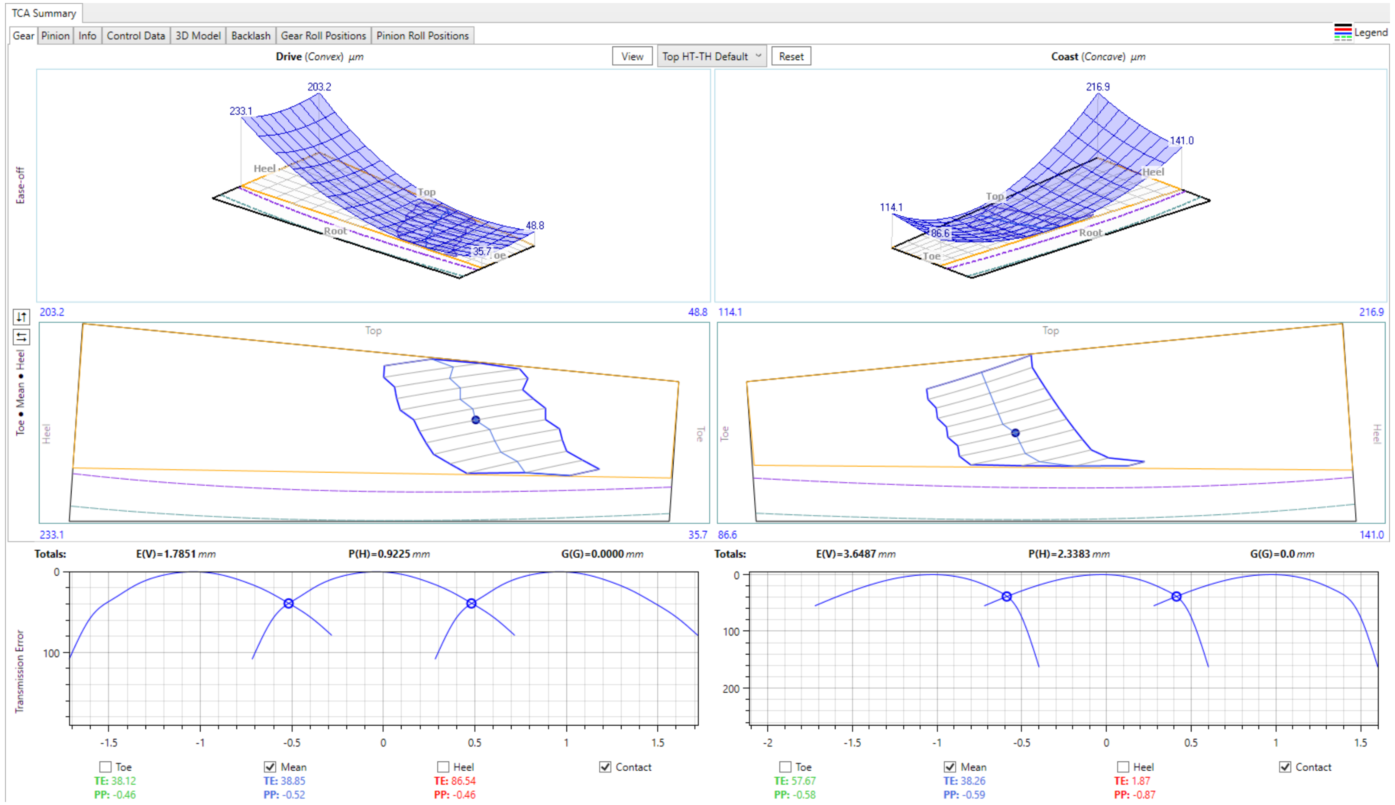
4. Sample design – Development of contact pattern

In the bevel gear software, the no load contact pattern is developed <u>without</u> considering misalignment values.

The contact pattern is pre-positioned to the toe.

The lengthwise crowning is designed for the nominal torque to reach 80% contact length under load.

A moderate profile crowning and flank surface bias are applied. The motion error of 40 µrad under zero load condition is achieved.



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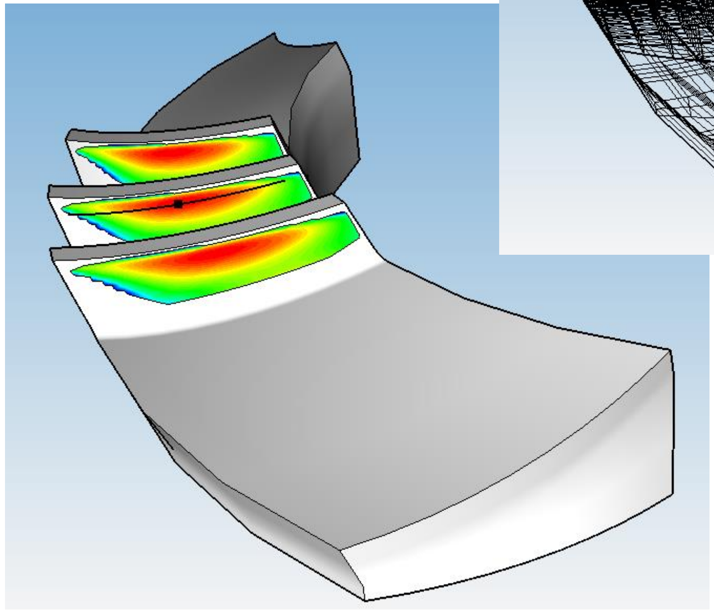
4. Sample design - Loaded tooth contact analysis

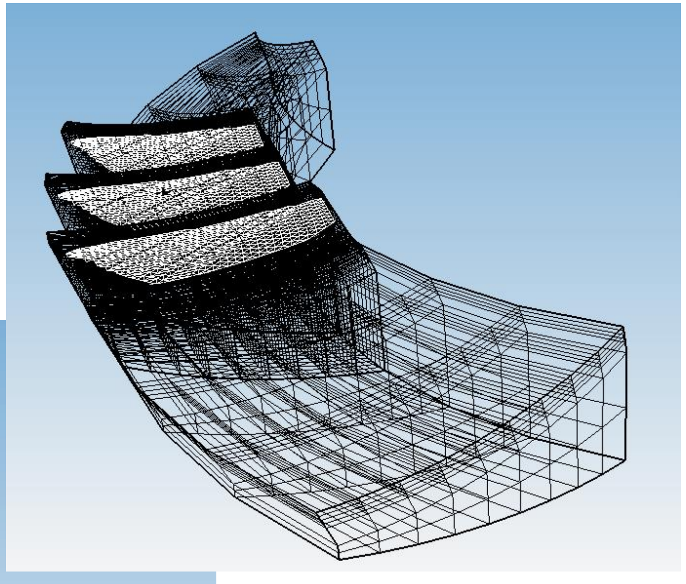
For the LTCA analysis, the GEMS® FEA App is used. This is a strength analysis program based on the finite element method.

The mesh generation is based on a 3D 8-node structure solid element type.

GEMS uses the machine settings approach to calculate the flank topology. It also allows to consider the misalignment values E, P, G and Alpha.

The evaluation includes a large variety of stress analysis options, as well as the transmission error under load.





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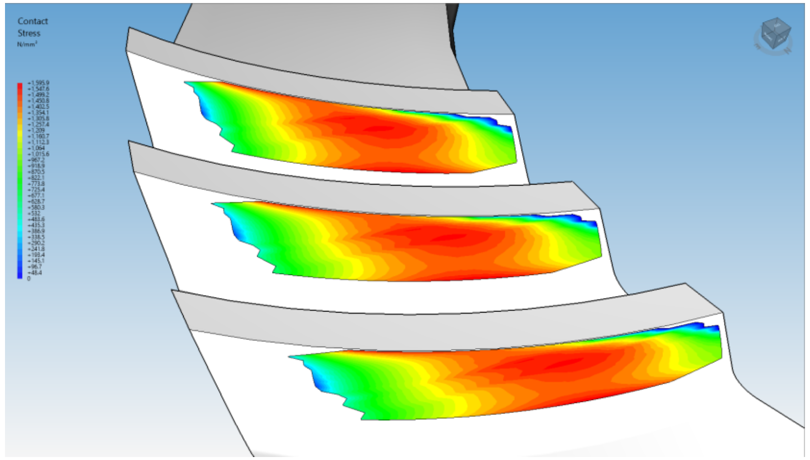
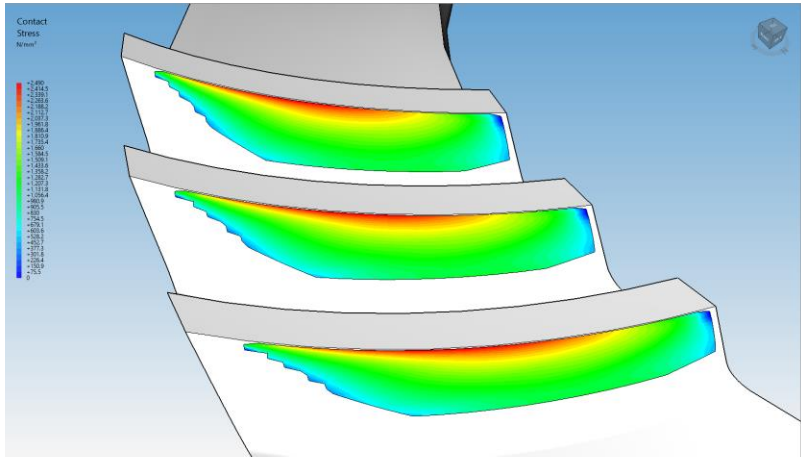
4. Sample design – Development of contact pattern

Under load, the contact pattern shows an ideal position, if the misalignments are ignored.

With the misalignment values E, P, G and Alpha considered, the contact pattern moves clearly towards the tip. This requires further contact pattern optimization.



Nominal load,
misalignments not considered



Nominal load,
misalignments considered

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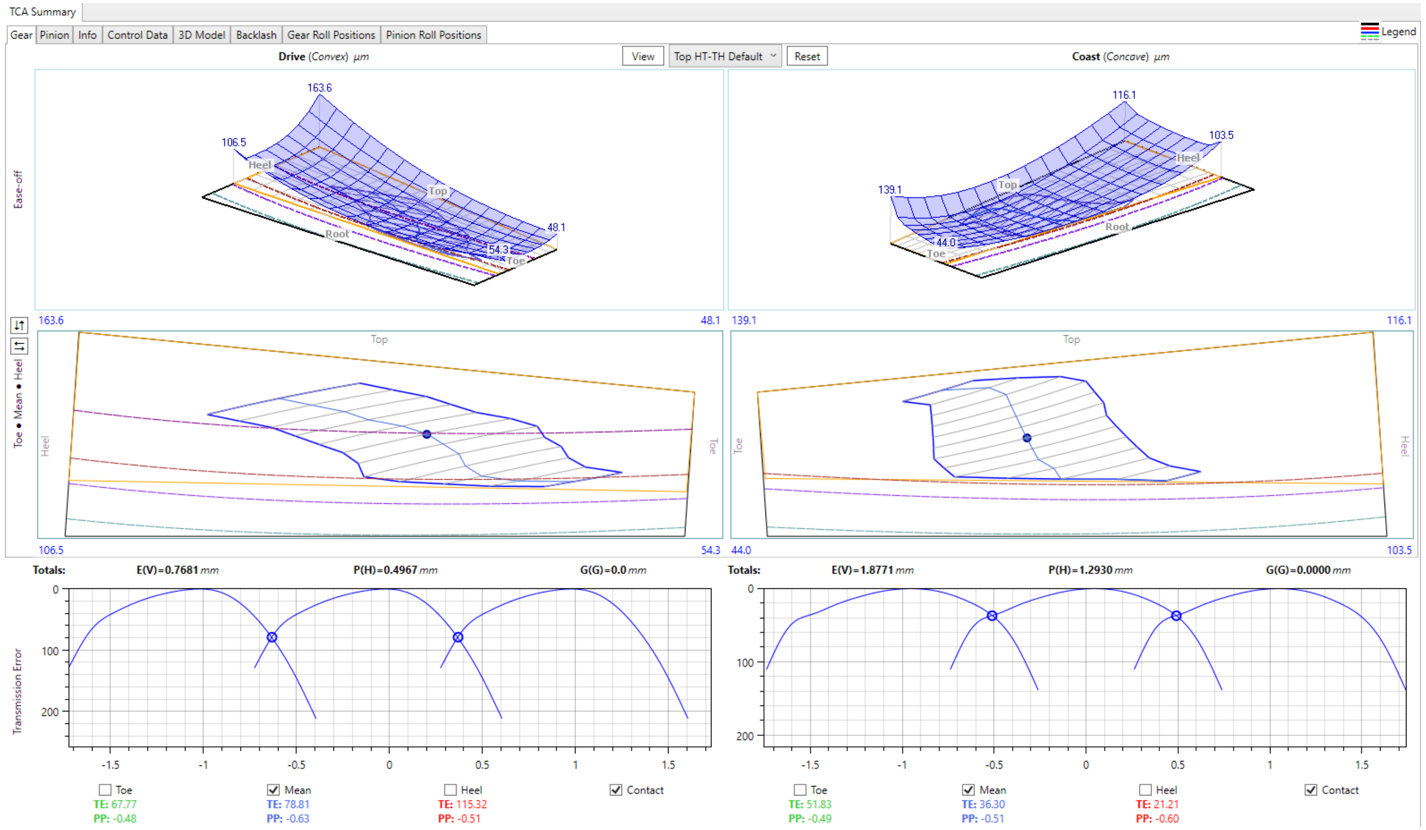
4. Sample design – Optimization of contact pattern

In a second step, the contact pattern is optimized considering the E, P, G and Alpha values.

In facewidth direction, the contact pattern is developed for largest spread.

To achieve a centered tooth contact in profile direction, additional reliefs in form of Flankrem on the ring gear and Toprem® on the pinion are applied.



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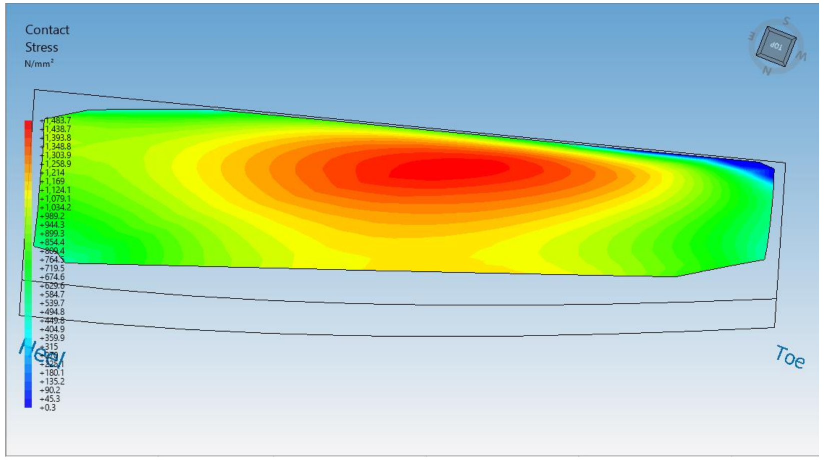
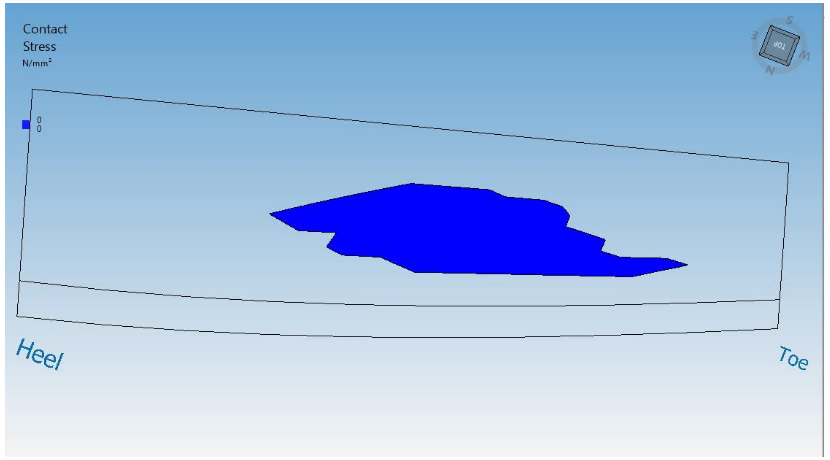
4. Sample design – Optimization of contact pattern

The contact pattern is checked at nominal load and additionally verified under light load (bench contact), to ensure that edge contact is avoided.

The optimized tooth contact pattern shows a good compromise between light load and nominal load.



Nominal load,
misalignments considered



Light load,
misalignments not considered

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4. Sample design - Stresses calculated by LTCA

The stresses calculated with LTCA are affected by the contact pattern position.

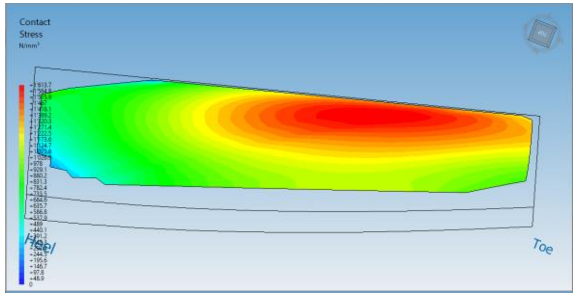
For various mounting distances of the pinion, the contact stress and pear-to-peak transmission error (PPTE) are evaluated.

The minimum contact stress was achieved with ΔH = -0.1 mm. But this contact pattern shows edge contact and increased peak-to-peak transmission error (PPTE), which is not acceptable.

Mounting Distance ΔH = +0.1 mm

Contact stress = 1614 N/mm2

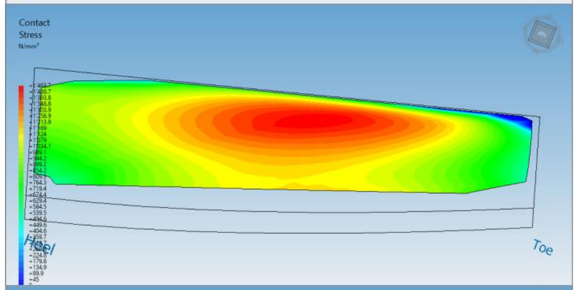
PPTE = 306.6 µrad



Mounting Distance ΔH = 0 mm

Contact stress = 1484 N/mm2

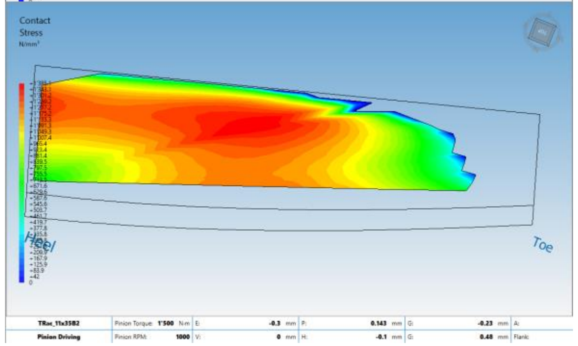
PPTE = 167.2 µrad



Mounting Distance ΔH = -0.1 mm

Contact stress = 1385 N/mm2

PPTE = 360.2 µrad



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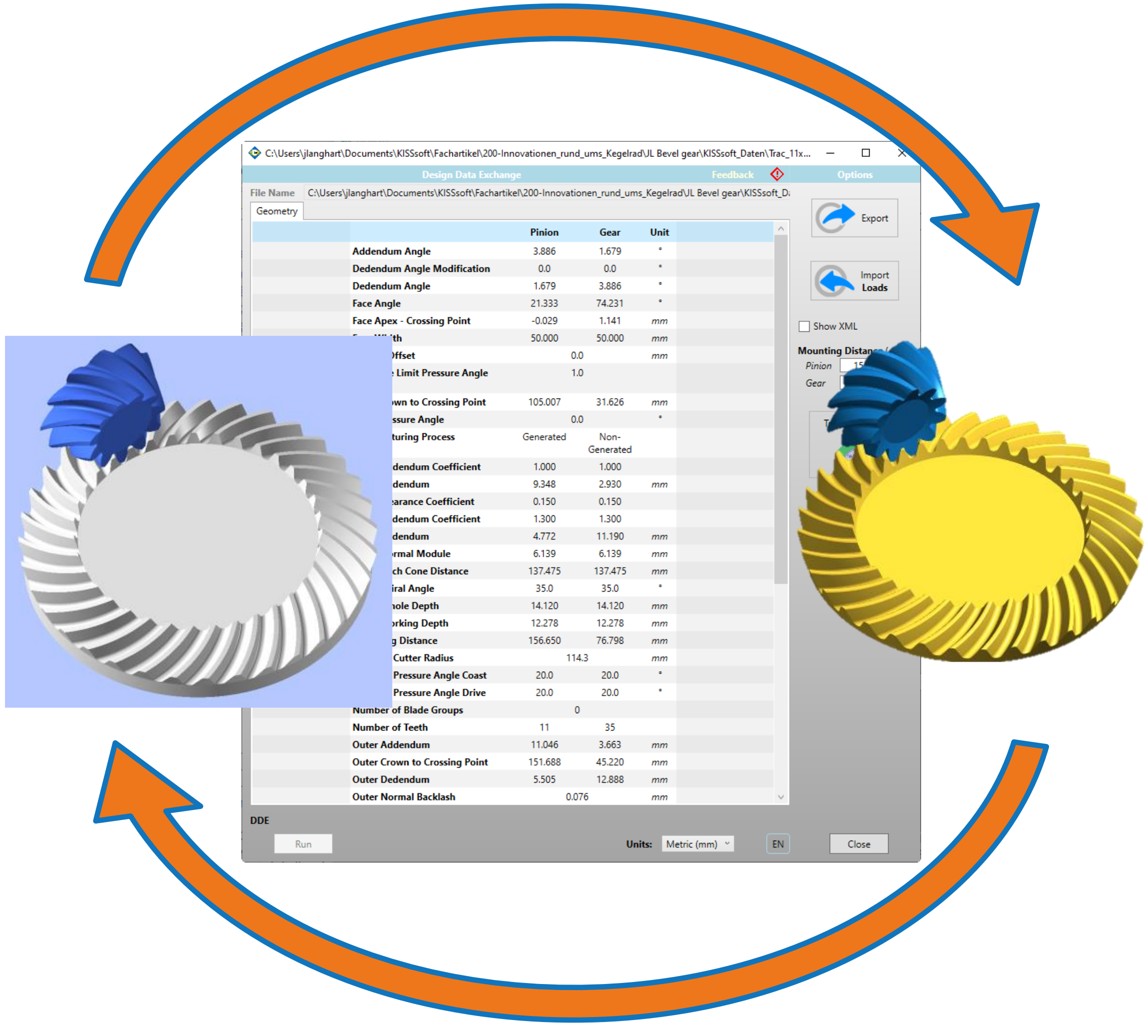
4. Sample design – Bevel gear calculation with ISO standard

For a comparison of stresses between the LTCA and the ISO standard, the equal geometry is mandatory between both software. Among others, it is:

- Tooth thickness on pinion and ring gear (“Equal life”, ‘Equal stress”, ..).
- Root radius (developed blade tip radius).
- … and many more.

To avoid errors, the data exchange per interface is recommended. The micro geometry is not transferred.



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4. Evaluation of stresses - Stresses by ISO standard

For the calculation of the stresses according to ISO 10300, the settings are:

Effective face width b_{eff}

Measured from the contact pattern in GEMS with 0.95.

Load distribution Z_{LS}

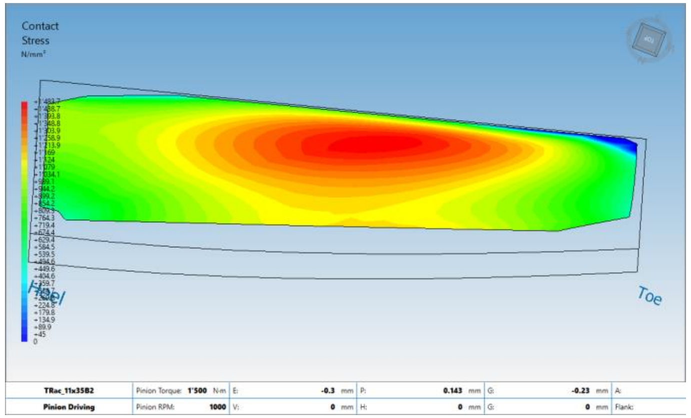
Profile crowning is considered as ‘low’.

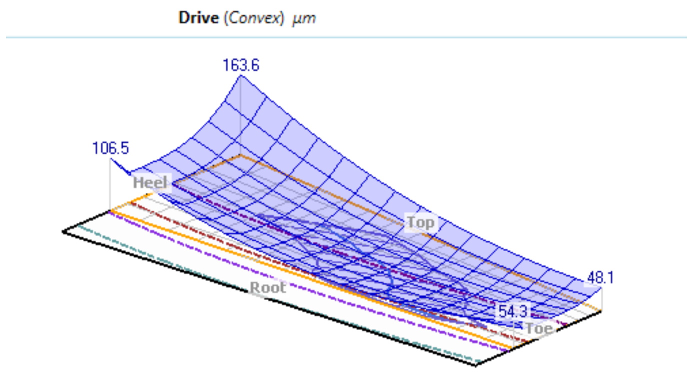
Face load distribution $K_{H\beta}$

Mounting factor $K_{H\beta\text{-}be}$ is = 1.1, which results in $K_{H\beta}$ = 1.65.

Other overload factors

For comparison with LTCA, the factors K_A , $K_{H\alpha}$ and K_V are = 1.





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5. Comparison of stresses - Root bending stresses

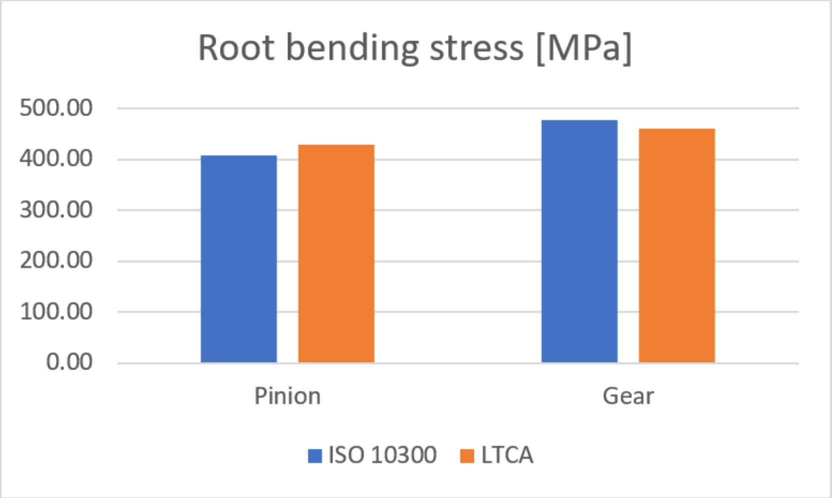
Root bending stresses

The root stresses are in mean value very similar between the ISO standard and the LTCA.

For the pinion, the stresses are slightly larger, for the ring gear slightly lower in the LTCA than in the ISO standard.

This is basically a good result, considering the fact, that the standard counts the root stresses at the 30º tangent, whereas the LTCA may not have the maximum stresses at the same point.

Root bending stress [MPa]		
	Pinion	**Gear**
LTCA (GEMS)	430.0	461.0
ISO (KISSsoft)	409.0	478.0
Difference (LTCA = ref)	**4.9%**	**-3.7%**



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5. Comparison of stresses – Contact stresses

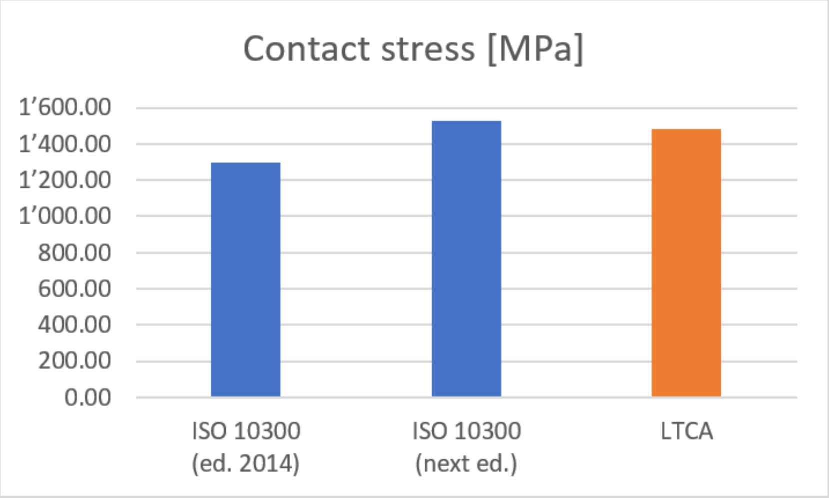
Contact stresses

The contact stresses show the difference between the current edition (2014) and the draft for the new edition of the ISO standard.

The contact stress according to the new edition matches much better with the LTCA calculation than the current edition.

This doesn’t harm the safety number for contact stress, as the permissible stress will be modified too.

Contact stress [MPa]		
	Pinion and Gear	Difference
LTCA (GEMS)	1’483.0	
ISO, ed. 2014 (KISSsoft)	1’300.0	**12.3%**
ISO, next ed. (KISSsoft)	1’529.4	**-3.1%**



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6. Evaluation with load spectrum – Safety numbers

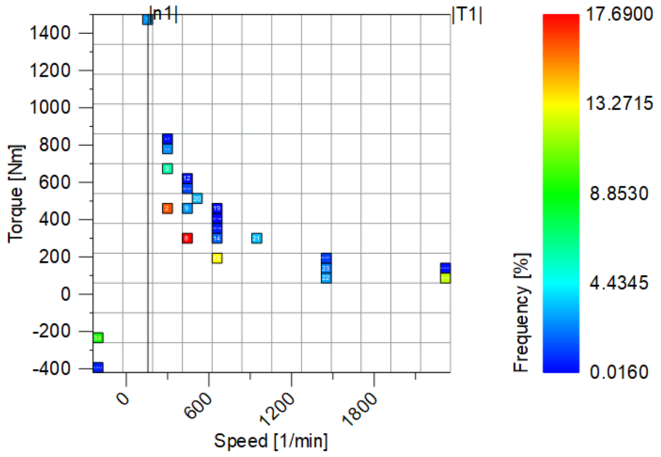
A load spectrum allows a more detailed definition of the operating load. The load spectrum was created using all the 8 speeds, including the reverse speed. Totally 36 load bins are applied.

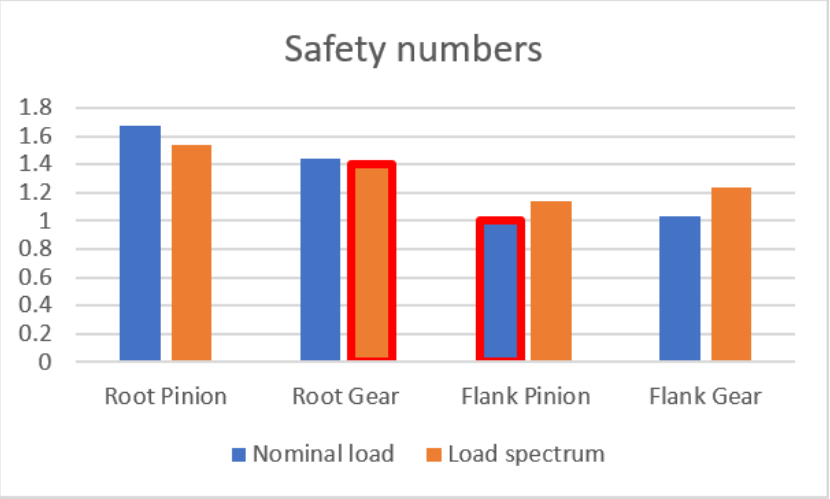
The load spectra calculation shows two points, which differ from the calculation with nominal load:

a) The maximum transmittable torque applicable is different:

- Maximum torque for nominal load: 1’651 Nm
- Maximum torque for load spectrum: 1’943 Nm

b) The safety number for contact stress is higher, as the reverse speed applies for the coast side.





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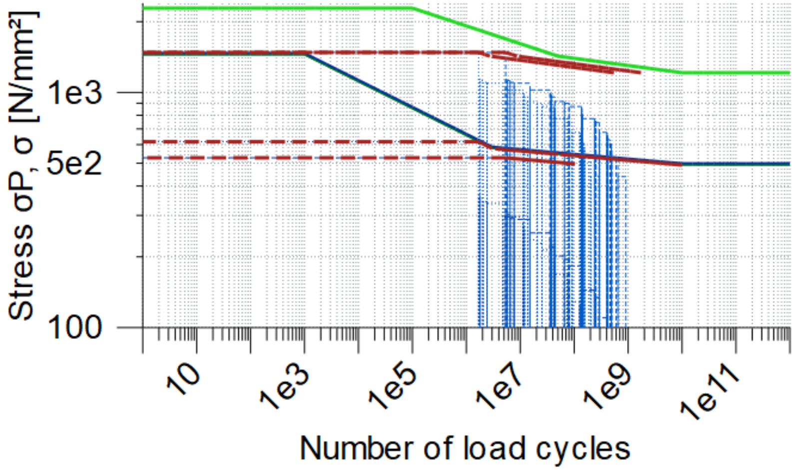
6. Evaluation with load spectra - Damage accumulation

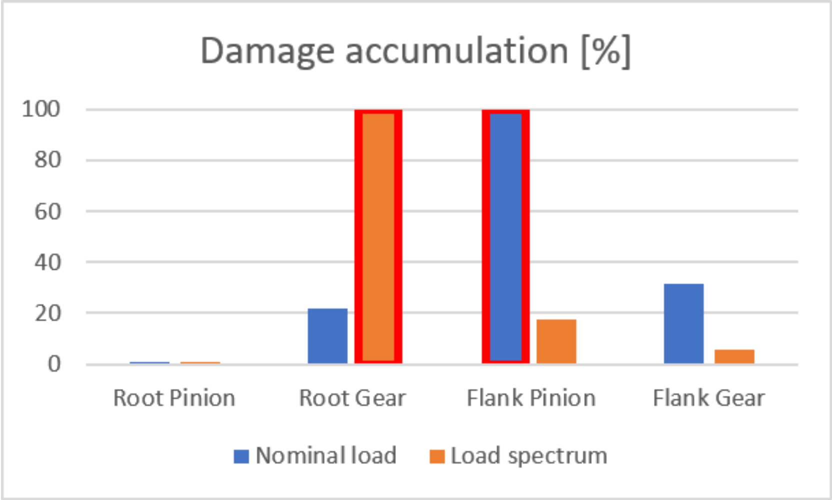
The **Wöhlerline** gives a good understanding for:

- In what area of the slope for permissible stress are the loads?
- Will any change in the stress level have a large impact on the lifetime?

The **damage accumulation** gives a more comparable number for the individual failure mode than the safety numbers.

- Highest damage with nominal load: Pitting on pinion.
- Highest damage with load spectrum: Root breakage on ring gear.





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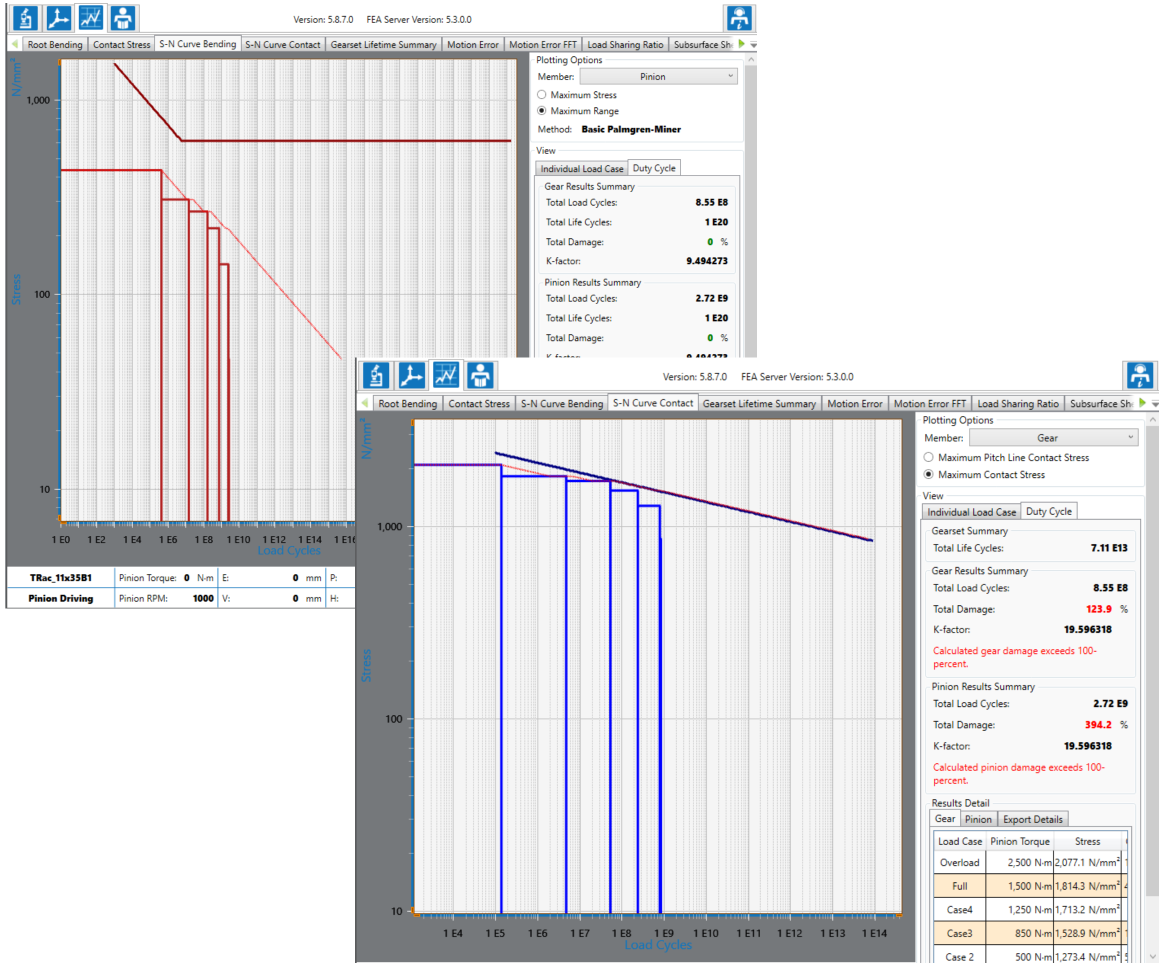
6. Evaluation with load spectra – Rating in LTCA tool

With specific gear analysis software, it is possible to integrate also the calculation of permissible stresses.

This allows an evaluation of the lifetime directly within the LTCA tools – including the corresponding E, P, G and Alpha values for all load cases or mounting distances.

It is important, that the rules of the specific standard are consistently implemented for the calculation of stresses but also permissible stresses.

The Wöhler lines can be defined depending on the preferred criteria.



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6. Conclusion

The rating of bevel gears is available by standards or by the LTCA method. The rating by standards is very quick and allows to find the best macro geometry by e.g. variant calculations.

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Modern gear analysis software can combine the stresses calculated by LTCA and the permissible stresses and allow an overall rating of the bevel gearset.

***

The root stresses calculated by the rating standard match well with the LTCA. The contact stress calculation will be modified in the next edition of ISO 10300 and will match closely with the LTCA results.

***

The load spectrum allows a more accurate analysis of the bevel gearset than the nominal load. This affects the maximum transmittable torques, but maybe also a different failure mode for the bevel gearset can be found.

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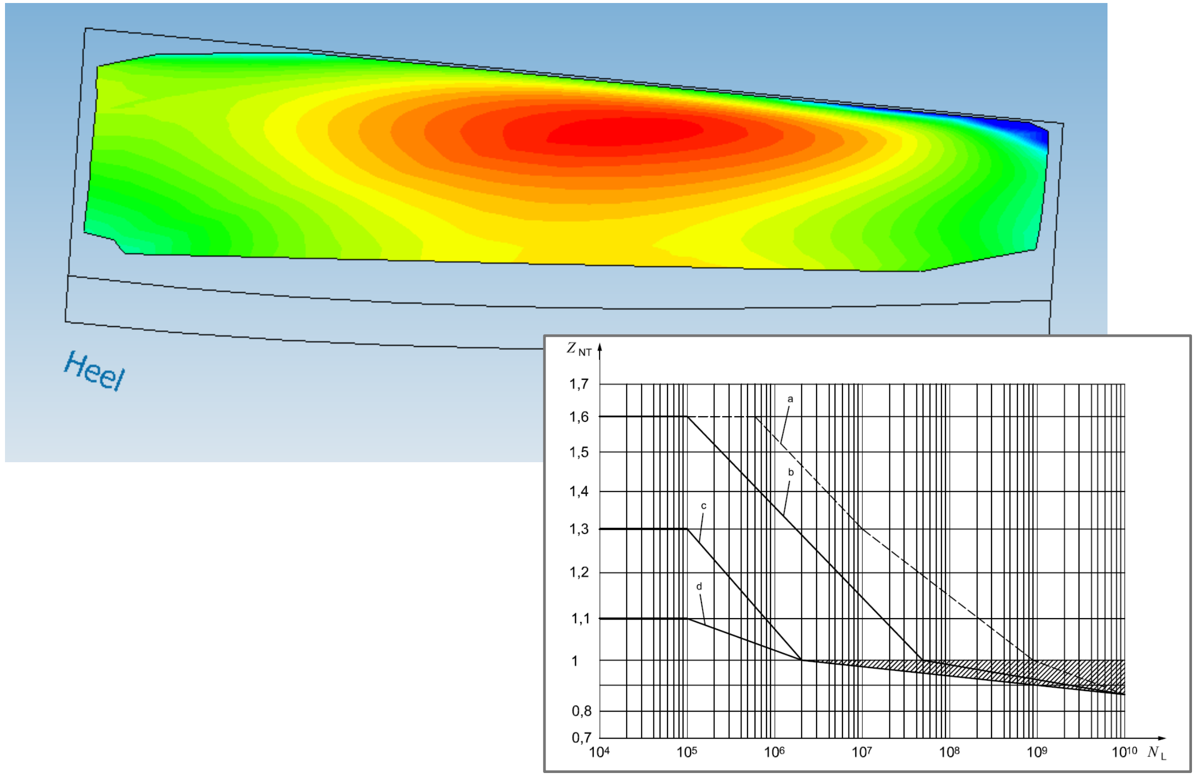
Thank you for your attention!



Sharing Knowledge

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